## A Close Call: How a Near Failure Propelled Me to Succeed

### Terence Tao

For as long as I can remember, I was always fascinated by numbers and the formal symbolic operations of mathematics, even before I knew the uses of mathematics in the real world. One of my earliest childhood memories was demanding that my grandmother, who was washing the windows, put detergent on the windows in the shape of numbers. When I was particularly rowdy as a child, my parents would sometimes give me a math workbook to work on instead, which I was more than happy to do. To me, mathematics was an activity to do for fun, and I would play with it endlessly.

Perhaps because of this, I found my mathematics classes at school to be easy—perhaps too easy—even after skipping a number of grades. If a lecture was on a topic I found interesting, I would use the class time to experiment with the material, perhaps finding alternate derivations of some step the teacher did on the board, or to plug in some numbers to try out special cases and look for patterns. If instead I found the topic to be dull, I would doodle like any other bored student. In either case, I did not take particularly detailed notes, nor did I ever develop any systematic study habits. I would be able to improvise my way through my homework and exams, for instance, by cramming through the textbook a few days before a final exam and perhaps playing a bit more with the parts of the class material that I really liked. It tended to work fairly well all the way up to my undergraduate classes. The courses that I enjoyed, I aced; classes that I found boring, I only barely passed, or (in two cases) failed altogether. (One class was a FORTRAN programming class in which I had refused to learn FOR-TRAN on the grounds that I already knew how to program in BASIC; the other was a quantum mechanics class in which we were warned well ahead of time that the final exam would require us to write a short essay on the history of the subject, which I totally ignored until the day of the exam, during which I still recall having to be escorted from the examination room in tears.) Despite this, I ended up graduating from my university with honors at the top of my class—but it was a small university with a tiny honors program, and in fact, there were only two other honors students in mathematics in my year!

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When I entered graduate study at Princeton, I brought my study habits (or lack thereof) with me. At the time in Princeton, the graduate classes did not have any homework or tests; the only major examination one had to pass (apart from some fairly easy language requirements) were the dreaded "generals"—the oral qualifying exams, often lasting over two hours, that one would take in front of three faculty members, usually in one's second year. The questions would be drawn from five topics: real analysis, complex analysis, algebra, and two topics of the student's choice. For most of the other graduate students in my year, preparing for the generals was a top priority; they would read textbooks from cover to cover, organise study groups, and give each other mock exams. It had become a tradition for every graduate student taking the generals to write up the questions they received and the answers they gave for future students to practice. There were even skits performed (with much gallows humor) on hypothetical general exams with a "death committee" of three faculty that were particularly notorious for being harsh on the examinee.

I managed to brush off almost all of this. I went to the classes that I enjoyed, dropped out of the ones I did not, and did some desultory reading of textbooks but spent an embarrassingly large fraction of my early graduate years messing around online (having discovered the World Wide Web in my first year) or playing computer games until late at night at the graduate dormitory computer room. For my general topics, I chose harmonic analysis—which I had studied for my master's degree back in Australia—and analytic number theory. Feeling that analysis was my strong suit, I only spent a few days reviewing real, complex, and harmonic analysis; the bulk of my study, such as it was, was devoted instead to algebra and analytic number theory. All in all, I probably only did about two weeks' worth of preparation for the generals, while my fellow classmates had devoted months. Nevertheless, I felt quite confident going into the exam.

The exam started off reasonably well, as they asked me to present the harmonic analysis that I had prepared, which was mostly material based on my master's thesis and specifically on a theorem in harmonic analysis known as the T(b) theorem. However, as they moved away from that topic, the shallowness of my preparation in the subject showed quite badly. I would be able to vaguely recall a basic result in the field, but not state it accurately, give a correct proof, or describe what it was used for or connected to. I have a distinct memory of the examiners asking easier and easier questions, to get me to a point where I would actually be able to give a satisfactory answer; they spent several minutes, for instance, painfully walking me through a derivation of the fundamental solution for the Laplacian. I had enjoyed playing with harmonic analysis for its own sake and had never paid much attention as to how it was used in other fields such as PDEs or complex analysis. Presented, for instance, with the Fourier multiplier for the propagator of the wave equation, I did not recognise it at all, and was unable to say anything interesting about it.

At this point, I was saved by a stroke of pure luck as the questioning then turned to my other topic of analytic number theory. Only one of the examiners had an extensive background in number theory, but he had mistakenly thought I had selected algebraic number theory as my topic, and so all the questions he had prepared were not appropriate. As such, I only got very standard questions in analytic number theory (e.g., prove the prime number theorem, Dirichlet's theorem, etc.), and these were topics that I actually did prepare for, so I was able to answer these questions quite easily. The rest of the exam then went fairly quickly as none of the examiners had prepared any truly challenging algebra questions.

After many nerve-wracking minutes of closed-door deliberation, the examiners did decide to (barely) pass me; however, my advisor gently explained his disappointment at my performance, and how I needed to do better in the future. I was still largely in a state of shock—this was the first time I had performed poorly on an exam that I was genuinely interested in performing well in. But it served as an important wake-up call and a turning point in my career. I began to take my classes and studying more seriously. I listened more to my fellow students and other faculty, and I cut back on my gaming. I worked particularly hard on all of the problems that my advisor gave me, in the hopes of finally impressing him. I certainly didn't always succeed at this—for instance, the first problem my advisor gave me, I was only able to solve five years after my PhD—but I poured substantial effort into the last two years of my graduate study, wrote up a decent thesis and a number of publications, and began the rest of my career as a professional mathematician. In retrospect, nearly failing the generals was probably the best thing that could have happened to me at the time.

My write-up of my general exams experience is still available online. I have been told that it has been a significant source of comfort to the more recent graduate students at Princeton.



Terence Tao

#### **Credits**

Author photo is courtesy of Reed Hutchinson/UCLA.

# My Journey from Slippery Rock to Duluth

### Joseph Gallian

My journey from Slippery Rock State College in Pennsylvania in 1966 to the University of Minnesota Duluth in 1972 had quite a few twists and turns. In 1966, I received TA offers from Minnesota, Kansas, Purdue, and Michigan State. About a week before the deadline for accepting offers, I selected Minnesota. A few days later, Kansas called to ask me if I would be willing to come to KU on a five-year NASA Fellowship. I was delighted to accept.

Because the Rock was almost exclusively a school for the preparation of K-12 teachers, my course work there did not adequately prepare me for graduate-level courses. Instead, I took courses intended for juniors and seniors. Fortunately, I had a charismatic abstract algebra teacher named Lee Sonneborn for both semesters. I was so enthralled with that course that I took an independent study course in permutation groups and participated in a weekly seminar on infinite group theory. By the middle of my second semester, I decided to do a PhD thesis on infinite groups under Sonneborn. This plan abruptly changed a few months later when Sonneborn moved to Michigan State. Disappointed, but not deterred, I decided that I would do a thesis with Dick Phillips, who was another infinite group theorist participating in the infinite group theory seminar. But it was not to be. In the fall of 1967, Phillips told me that he would be going to Michigan State the next year.

The departure of both of my potential thesis advisors prompted me to apply to grad school at Michigan State, Utah, Illinois, and Notre Dame, all of which had infinite group theorists. When I received a three-year fellowship from Notre Dame, I accepted. Shortly after arriving at Notre Dame, I approached the infinite group theorist about working with him. As luck would have it, he told me that this was his final year at Notre Dame. The next best option was to work with Warren Wong, who was one of hundreds of people working on the classification of finite simple groups. Wong agreed to take me on, but he said he would be on sabbatical the next year in New Zealand and I was welcome to join him there. This did not appeal to me and my wife, so I declined. That left me with three options: transfer, abandon group theory, or do a thesis with Karl Kronstein, whose only publication was on representations of finite groups, a subject about which I knew nothing. I opted for the last.

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