Math as Social Endeavor: Groupwork and the Blackboard

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We are teaching at a time when the benefits of active learning and group work in introductory STEM are well established and evangelized, with some schools pursuing a broad transition to a flipped classroom environment [Mc], and others increasing the prevalence of peer-led team learning (PLTL)[1] workshops [Fr1]. The original purpose of this article was to examine successful modes of group work at many levels of mathematics, and how the peculiar nature of our subject's discourse favors the blackboard as a means of communication and collaboration.

Once we began to write, however, it emerged that the menace of COVID-19 would force universities worldwide to shift abruptly to online instruction, posing obvious difficulties for decentralized learning environments and blackboard lectures alike. Contemplating the magic of a physical gathering of people, with a box of chalk and a wall of enamelled steel, is a wistful exercise amidst this giant, painful social distancing experiment; but perhaps it suggests some ways in which we can better configure our virtual courses, discussions, and seminars in the meantime.

As a social construct that weaves together the products of many individual imaginations, mathematics benefits from being communicated on a large canvas, so that the imaginative scope and the patterns of the weave can be conveyed and recognized (whether it is one or many doing the writing). In addition, there is a particular welding of conceptual metaphor and symbolic precision which makes our endeavor cognitively feasible, and which for two centuries has evolved along with the use of blackboards to convey our ideas.

The social construct aspect of mathematics makes it like learning a foreign language. One doesn't succeed in this by listening to an instructor and doing grammar exercises alone. There must be an active component, which, short of a sojourn in the foreign capital (Research Mathematics, pop. 100,000?), is the conversation portion of such a class.

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[1]*A collaborative learning environment in which roughly ten students, led by a trained "peer leader" who has mastered the material, work together on a packet designed to build problem-solving skills for a specific course.*

The analogue for math is problem solving in groups and student presentations.

Math education research indicates that the cooperative learning fostered by group work significantly increases motivation and performance in introductory mathematics courses, and reduces attrition from STEM majors [Ku]. The discussion sections for such courses are the natural home for such group collaboration, ideally centered around a shared blackboard or whiteboard. Work at group whiteboards also lends itself to quick presentation of solutions to the rest of the class, and to integration with PLTL, by distributing team leaders among the group. In both undergraduate and graduate courses that lack a discussion section,[2] the students can still benefit from group work if the instructor is willing to set aside some lecture time each week.

In small-group settings—from the summer calculus course to graduate learning seminars—assigning problems or readings for students to present can be useful or even transformative. The reader is doubtless familiar with the mental scaffolding that is raised by actually teaching a subject. It is no accident that the best advisors are skillful listeners: P. Griffiths, who himself excelled at the technique, relates how his advisor D. Spencer would invite him to read and present an article on the board, and critique the presentation [St]. We hasten to add that this approach relies on the willingness of students, and "getting them up to the board" has not always been popular. Upon the introduction of blackboards at Yale, students were required to draw conic sections on them as part of an exam; a rebellion ensued which necessitated expelling nearly half of the class of 1832. A less dramatic mutiny forced the second author to abandon group work in a UK tutorial class he taught: the students, who had been working away on a blackboard (each group to a physical panel) rather than sitting and copying a lecturer's solutions as usual, complained to the maths office that they departed with incomplete notes.[3]

Having extolled the virtues of doing math in groups, let us now throw cold water on the notion of scrapping lectures. The lecture conveys the imaginative aspect of math: the overarching narrative which explains the development of concepts, the patterns relating them, and the connections to other subjects. It disabuses students of the notion that mathematics is a sequential discipline, in which the right steps can be applied to the right problem in the right order to arrive at the correct solution, and supports the more explicit forms of active learning by demonstrating the rounded form of reasoning that we expect them to develop in their group work and showcase in their presentations. The contour of the lecturer's reasoning, superimposed on

[2]*In the UK, some math departments curate problem sheets for tutorials (i.e., discussion sections) at all course levels; this produces much better results than pulling problems out of a textbook. However, they do not generally have the students work in groups.*

[3]*The iPhone did not yet include a camera.*

the typically linear organization of a textbook or article, gives a glimpse at some of the different lines of thought that are possible—some fruitful and some not, all woven into what simultaneously solves a problem and prods at the question of what the mathematics means and where its application makes sense. There is also beauty: the frisson of excitement from showing a class Newton's triumph in explaining Kepler's Laws is one of the most rewarding parts of teaching calculus. Blackboards remain the superior technology for communicating all of this, not least because they slow the speaker to a pace commensurate with human cognition.

Rather than flipping lectures, we should train our students to approach them, as we do, as a form of active learning. When we as mathematicians want to understand someone's work, we get them in front of a blackboard for a talk and pepper them with questions throughout, because it's effective and efficient. (Try doing that with a prerecorded lecture!) University teaching centers can help instructors better facilitate this kind of interaction through the use of OPAL[4] [Fr2] and TRU[5] [Sc] lecture observations.

As we temporarily forego our symbiotic relationship with blackboards and chalk, it is worth thinking about how these tools influence the performance and communication of mathematics, and what we might use as their virtual surrogates. The sheer area framed in a blackboard invites creativity and collaboration by erasing the intimidating natural starting point and linear progression imposed by a narrow sheet of paper. It allows one (or many) to organize thoughts spatially, giving each theorem and conjecture a physical location, connected by the arrows of reasoning like a television detective's case board.

In the scramble to move classes online, many universities have turned to the video-conferencing app Zoom, whose screen-sharing function allows speakers to display whatever note-taking app they prefer.[6] The effect, more akin to a document camera than a blackboard, does miss some of the illustrative power afforded when the board and participants are embedded in a single space. Its higher potential for collaborative modes may offer some compensation: others can annotate the screen using Zoom's whiteboard technology, which can also be used on its own in breakout rooms for group work.[7] We must try to make the experience as dynamic as possible, and find a new social construct for the age of the virophobic telecommute.

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[4]*Observation Protocol for Active Learning (OPAL) is designed to provide lecturers with a snapshot of activities and interactions in a large lecture class.*

[5]*The TRU (Teaching for Robust Understanding) Observation Guide is more specific to mathematics, and provides five criteria for measuring classroom activity.*

[6]*For prepared lectures, an interesting option is the software application Prezi, which allows one to organize the contents of a talk on a single giant slate that one can move around and zoom in and out of.*

References

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[7]*Unfortunately, the whiteboard is at present too coarse to be useful for lectures. Help us, Zoom developers!*