expressed not only a deeper understanding of the types and uses of mathematics, but also a resurgent enthusiasm for pursuing a mathematics degree as a result of their experience in All Girls / All Math.

By adapting to new media for learning and continuing to extend such opportunities to young women in STEM, camps like All Girls / All Math are giving them the opportunity to be able to learn alongside the COVID-19 pandemic and find new ways to apply their knowledge. A virtual summer camp has become another platform to promote an engaged, adaptive, and forward-thinking network of young women, poised to revolutionize the field of mathematics.

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Credits

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Targeted High School Math Competitions: Girls in Math at Yale

Elaine Hou and Noah Kravitz

Introduction

In 2017, the Yale Math Competition (YMC) undergraduate student organization founded a new initiative called Girls in Math at Yale. This event, which now occurs annually, serves high school girls in the local community and aims

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to promote an interest in and passion for math. Organizing and hosting Girls in Math at Yale has taught us a great deal, and we hope to share with you our insights into developing effective math outreach initiatives.

About Us

YMC was founded in 2014 as the Yale branch of the Math Majors of America Tournament for High Schools (MMATHS). The idea of college students organizing math events for like-minded high school students was not original—HMMT, PUMAC, SMT, and CHMMC, for instance, are older—but what makes MMATHS distinctive is its geographical diversity. The MMATHS annual competition is hosted simultaneously at university sites in different states, and the long-term goal is to have at least one participating university in every state of the US. So far, we have five participating schools: University of Florida, Columbia, Yale, University of Michigan, and University of Virginia. We have quite a ways to go!

Most of the organizers of YMC are veterans of various high school math competition circuits, such as the AMC/ AIME/USAMO series and ARML. Many of us also took part in the Putnam Competition during our time at Yale. Participating in these events not only cultivated our interest in math but also introduced us to a bustling community of rivals, peers, and friends.

Motivation and Philosophy

Although Girls in Math at Yale began in 2017, YMC had been discussing the idea of holding such an event since its founding three years earlier. Many of our female members could directly link their decisions to pursue STEM majors and careers to formative previous experiences in women's initiatives such as Math Prize for Girls. Since there are relatively few of these influential outreach activities, we saw the need for a new girls' event and realized that YMC was in a perfect position to make this happen.

We frequently debated the merits of hosting a girls' event. Some members pointed out that hosting a competition specifically for girls inadvertently sends the



Figure 1. Participants collaborate on a team round.

message that girls need special treatment. Other members responded by noting how encouraging girls' events can be. Our female classmates and mentors reinforced the idea that being a minority member of a community—the only girl in a math class or the only female professor in a math department—is itself a frustrating and isolating experience. We all agreed on at least one worthwhile reason to hold a girls' event: to show young women that they are not alone.

It is important to develop a guiding philosophy before crafting an outreach event for female students (or any other group, for that matter): who is the event for, and what is its goal? The YMC team identified three themes that drove the development of Girls in Math at Yale. First, a math outreach event should encourage a lasting interest in and appreciation of math. It should give people an opportunity to celebrate their improvement and achievement, and it should be satisfying and enjoyable, so that participants look forward to future events. Second, a math competition must be intellectually stimulating. Problems should spark discussion and insight, and additional components, such as plenary speaker sessions and undergraduate panels, should show students where math can take them. We want participants, teachers, and coaches to feel enriched-that the event was well worth their time. Third, a math outreach event should reflect the collaborative nature of modern mathematics. As in all STEM fields, mathematical breakthroughs often come about through teamwork. We want participants to mingle, engage with one another, and feel like part of the math community.

How to Organize a Math Competition

Running any math competition requires a lot of work. Planning, which usually takes many months, has two main components: organizing the event and preparing the mathematical content. We at YMC have found the following approaches and rules of thumb helpful for all of our high school events.

Logistics

Like organizing a conference, organizing a math competition involves many logistical details. But fret not: the workload is manageable with careful planning. And, of course, the benefits for the participants are well worth the effort. Here, we highlight the key logistical steps that go into holding a math competition:

- *Set clear goals* for the event, including length and number of participants.
- *Select a date* for the competition well in advance. Try to avoid scheduling conflicts with other competitions and standardized testing dates.
- *Book a venue.* Pick a location that will work well for the activities you want to hold: if you have a team component, students will need places to sit and converse in small groups; if you have a speaker, an auditorium will be ideal.
- Advertise the event by reaching out to students, schools, and extracurricular organizations. One



Figure 2. Participants compete in an individual round.

way to identify contacts is to see which schools or organizations already participate in math competitions. If this is your first time hosting a math competition, you may need to send some cold emails. (Recruitment gets easier over time.)

- *Register participants* by collecting permission slips, waivers, disclaimers, photo release forms, and fees. Select a registration deadline and notify your potential recruits of this deadline.
- *Secure resources and materials* for the event. All competitions require testing materials, and most provide awards. T-shirts and good food also make people happy. Sponsorships from academia or industry can help defray expenses. Reach out to potential guest speakers well in advance.
- *Recruit volunteers* to help on the day of the competition. They do both the literal and metaphorical heavy lifting: registering participants, proctoring and grading tests, setting up rooms, helping students find their way around, chatting with coaches, and cleaning up at the end of the day. Host an information session for your volunteers so they understand their roles, and entice them with free food and T-shirts. At MMATHS, participants wear T-shirts of one color and volunteers wear T-shirts of another color, for easy identification.
- *Create a schedule* for the competition. Keep in mind that competitive portions need to take place early enough that you can finish grading before the awards ceremony. Our rule of thumb is to budget 15 to 30 minutes extra for each event—this buffer time will help you stay on schedule.
- *Host the event.* Congratulations! You're hosting a math competition! Every competition buzzes with excitement. It will be hectic, and that is okay. Have fun, and don't forget to take pictures!
- Send out post-event materials. Inform participants, schools, and organizations of their performance

at the event, and send out a post-event survey. Ask people what went well and what can be improved.

Problem-writing

The mathematical preparation consists of crafting tests. MMATHS and Girls in Math at Yale each feature a combination of individual and team rounds. The tests do not serve only to determine winners—much more important is that participants have fun and come away having learned something.

What makes for a good problem? Above all else, it should be inherently interesting, in the sense that one would be curious about its solution even without the incentive of scoring points. A problem is interesting if it makes a high school test-taker say, "Oh, that's cool!" Both hard and easy problems can be interesting. (An easy interesting problem may require only a simple insight for its solution, whereas a difficult one could call for a touch of divine inspiration.) Much like choosing a good research topic, identifying an interesting competition problem is a subjective matter of aesthetics. The most interesting problems emphasize onthe-spot cleverness and incorporate uncommon themes. Write a problem about polynomial coefficients where the solution doesn't just follow from an application of Viète's Formula. Ask a question about a wacky new number-theoretic function that none of the students has seen before. Replace Fibonacci and Catalan numbers with more exotic variants.

Designing good problems is an art. The following strategies can help you get started. First, start with a question (perhaps from real life) that you genuinely don't know how to answer. If you're wondering about the solution, then it's probably interesting to other people, too. You can sometimes make the original question nicer by tweaking a parameter or considering a variation. Wondering how much you can tilt your glass of water without making a mess? Just assume that the glass is a perfect cylinder and fix an amount of water, and you can leave the rest to the high schoolers! Second, start with a key idea and reverse-engineer a problem whose solution uses this insight. Such a problem is guaranteed to generate a satisfying "aha!" moment. You can even disguise the key idea so that it appears only after an initial reduction. Amused by double-sum identities? Recast your favorite one as a problem about a sequence of triangle side lengths. Third, start with an advanced technique or result from college-level math or beyond. Even if this content isn't appropriate for high schoolers, a simplification or special case might be tractable. Isolating the crucial step and recasting it in elementary terms adds a touch of "deep math." Intrigued by the arithmetic derivative? Give a quick definition and then ask for the number of fixed points in an interval of natural numbers.

Most of all, just write lots and lots of problems. This includes bad problems. Writing problems in any capacity gets your creativity flowing, and some of your problems are bound to be good if you produce enough of them. It's worth holding onto all draft problems since you might later see ways to salvage partial ideas. Finally, avoid common pitfalls that tend to lead to uninteresting problems: extensive computation (who wants to multiply six-digit numbers or check a dozen cases by hand?) and reliance on prior knowledge of tricks, techniques, and definitions.

What makes for a good test? The mark of a well-designed test is that it keeps a broad range of test-takers engaged. Budding algebraists, number theorists, geometers, and combinatorialists should each find a mix of familiar and unfamiliar material. Some students are attracted to word problems with cute set-ups, and others prefer cut-and-dry statements. In order to work productively for the entire allotted time, stronger participants need adequate challenges, and less experienced students need approachable problems-a test should appeal to test-takers at a wide range of levels. The key to achieving these balances is variety: variety in subject area, variety in mode of presentation, variety in difficulty. As a rule of thumb, the vast majority of the problems should feel accessible to the vast majority of the test-takers. If a hard problem has an unintimidating statement, then everyone can enjoy looking for patterns and attempting solutions, even if they're on the wrong track. A problem with both fast (clever) and slow (less clever) solutions also caters to a wide audience since each person can spend time on the problem and make progress as they are able.

Keeping an extensive problem database eases the test-compiling process. If you assemble many more problems than you need, then you have more flexibility in creating balanced tests. When you have two good problems that are too similar to appear in the same competition, save one for the future! Test-solving is the best way to determine whether an array of problems is balanced and interesting. Ask your friends (ideally, not the same people who wrote the problems) to take the tests and tell you what was fun, hard, frustrating, and so on.

How Girls in Math at Yale Is Different

Girls in Math at Yale required that we fine-tune our general math competition model in a number of ways. On the logistical front, the most important consideration was scale. The existing annual MMATHS competition involved over six hundred students spread across five sites in two different time zones (as well as about a hundred observant Jewish students for Yale's Sunday re-run of the main Saturday competition). By contrast, Girls in Math at Yale was a deliberately local, smaller-scale event—we wanted to create a comfortable, welcoming event free of the pomp and circumstance of large math contests. We had around sixty participants for the first Girls in Math at Yale, and that number has since climbed up towards a hundred. These parameters informed our registration outreach, volunteer recruitment, and venue planning.

We prioritized finding strong female role models for the high school participants. To this end, we reached out to Yale's various women-in-STEM groups in addition to the



Figure 3. High school students engage in conversations with Yale students over lunch.

general-interest STEM groups that provided most of the volunteers for MMATHS. On competition day, we took special care that the women on our team appear in prominent leadership roles, particularly in the opening and concluding ceremonies. One of the most special parts of the event was the informal pizza lunch with Yale students. (After the first Girls in Math at Yale, we received such overwhelmingly positive feedback that we doubled the length of the lunch for subsequent years.) Female Yale students in a wide range of STEM fields-everything from math and statistics to computer science, physics, and biology-chatted with small, rotating groups of high schoolers who shared their interests and wanted to learn more about their college experiences. This opportunity to connect with older role models spoke directly to the "inspiration" and "empowerment" aspects of our mission. This supplemental enrichment rounded out our program with a nice personal touch.

When we modified our MMATHS test format for Girls in Math at Yale, we converted the traditional team round of twelve standalone problems into a themed team round that emphasized learning and understanding new concepts. By choosing a topic that was unfamiliar to the participants and introducing it in a way that did not require prior knowledge, we deliberately leveled the playing field and helped more students gain the confidence to contribute. (Two of our most successful topics were permutation pattern containment and graph coloring.) We wrote the themed round in a narrative format, with problems of varying difficulty sprinkled throughout, and we encouraged the students to ask clarifying questions. The resulting organizer-participant dialogue made the test-taking experience feel less adversarial than in an ordinary math competition. Finally, the themed round format fostered collaboration, rather than competition, among participants. Because of the cumulative nature of the problems, team members had to work together and share their insights. Whereas a traditional team round usually devolves into a mad scramble to divvy up problems and find tricky "gotcha!" solutions (which can



Figure 4. Happy team members show off their well-earned awards.

be fun in other settings), our themed round encouraged measured consideration and teamwork.

Takeaways

Girls in Math at Yale caters to young women. Future events with the same model could effectively reach other people, such as members of racial minorities and students from under-resourced schools, who are underrepresented on the math competition circuit. When designing such an event, one must think carefully about both audience and goals: Who will the event serve, and how? Which aspects of the standard math competition model work well for this audience, and which should be modified? What will participants gain from the experience? And finally: How do we turn this vision into a reality?

Stay flexible, learn from your mistakes, and be open to radical revisions. What goes wrong the first time can be fixed the second time. Even if the final product isn't perfect, people will appreciate your efforts. We hope our example inspires new math outreach events!





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Credits

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