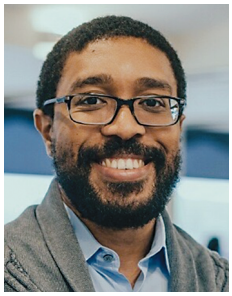
[She16] Margot Lee Shetterly, *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race*, Harper Collins Publications, New York, NY, 2016.

Ranthony A. C. Edmonds



John H. Johnson Jr.

Credits

Author photos are courtesy of JKE Photography.

Designing a Course Connecting Mathematics with Latin American Cultures

Colleen Duffy

We teach what we love; I love both mathematics and exploring other cultures. I had a strong desire to combine my interest and studies in Latin American cultures with mathematics, and hence the course entitled *Mathematics in Latin American Cultures* was born. Its course catalog description is:

> This course introduces important mathematical concepts and topics, such as number and arithmetic systems, symmetry, and data structure, using the cultural lenses of pre-Columbian Latin American indigenous cultures.

In this article we will explore how my interest in such a course developed, the logistics of designing a new course, and how to connect mathematical ideas with cultural components.

Background

I have always had a strong interest in exploring other cultures, peoples, and places. My family hosted an exchange student from Chiapas, Mexico, my senior year of high school, and she became a member of our family. Through several trips to visit her, I have been able to see first-hand several historical sites of the Maya peoples and to see some of how indigenous peoples today are connected to them. I double majored in mathematics and Spanish in university. During my semester abroad in Salamanca, Spain, I took a course on Spanish and Portuguese colonialism in Latin America. And, during graduate school, I went to Peru where I visited several Incan historical sites including Macchu Picchu. These experiences naturally led me to consider how I could incorporate a new perspective, and specifically an indigenous Latin American perspective, into a mathematics course. Furthermore, I have been able to enrich the course with stories and pictures from my travels. While my background prepared me for teaching a course based on Latin American culture, there are a lot of resources available to get you started in developing your own course if you do not have such a background. I mention some resources below.

Logistics

Our university and department espouses interdisciplinary collaborations in course work and research, so institutional support was not a concern in deciding to develop the course. At the end of my first year at the university I talked with the chair of my department to get his support in developing this course. I then found a colleague, Dr. Manny Fernández, in the Latin American and Latinx Studies (LAS) program who was interested in co-developing the course with me. It was invaluable to have his expertise on the cultural side of things, and fun to collaborate on ideas. We also brought two students into the project: a mathematics education major with a Spanish minor as well as an LAS major. We received financial support through our Office of Research and Sponsored Programs to pay the students a stipend to work on developing the course with us. Our university also now has funding for faculty to develop or revamp a course; so, it is worth checking into funding options. We developed it over the course of the year 2009–2010, and offered the course the following spring 2011.

The University of Wisconsin–Eau Claire, like many colleges, has a liberal education math course focused on introducing a variety of mathematical topics that the students have likely not seen before; here the course is entitled *Introduction to Mathematical Thinking*. This type of course provides a good platform to try a new course. There is a lot of paperwork involved in getting a new course in the catalog, so we decided to pilot our course as a special section of *Introduction to Mathematical Thinking* first. Because we have a lot of flexibility as to topics covered in the course, this was possible for us. Starting by introducing a project or a couple of topics in a course would also be a way to build up to designing a new course. Another possibility would be to offer a seminar or honor's course. Fernández and I co-taught the first two iterations of the course. Student interest was high enough that the mathematics department and the LAS program were interested in having it be its own course after that. It is housed in the math department

Colleen Duffy is an associate professor of mathematics at the University of Wisconsin–Eau Claire. Her email address is duffycm@uwec.edu.

DOI: https://dx.doi.org/10.1090/noti2215

(so I now teach it alone), but the course counts as elective credit in the LAS major or minor as well as satisfying our math requirement in the liberal education core. We get a variety of students in the course, not only students in LAS. We have offered one section a year since the course was first introduced.

Topics

In this section we highlight topics covered in this course, but also how to connect these mathematical concepts to cultural content in general. Most of these mathematical topics were chosen because of their natural occurrence in the culture being studied, but in some cases the mathematical topics were chosen and then the cultural connection was made. We also make an effort in the course to form direct cultural connections among the cultures we are studying, the modern cultures of those peoples and places, and the students' own culture.

The main mathematical topics covered in the course are: history of zero, base conversion, modular arithmetic, symmetry, groups, basic data structures, cryptography, Fibonacci sequence, and scaling factors. Throughout the course we also discuss more general topics such as the need for multidisciplinary collaboration, modern research in the area, and the need to analyze claims in light of both mathematics and culture. As stated above, the course uses the cultural lenses of pre-Columbian Latin American indigenous cultures to study mathematics. We decided to focus on the Maya, the Inca, and the Nasca.

Maya

The Maya were a Mesoamerican civilization that built on earlier cultures such as the Olmec and shared many traits with other peoples in the region. They used a vigesimal (base 20) number system. In the course we use this to introduce converting numbers between bases and doing arithmetic in other bases. We start with base 20, but we also discuss using other bases such as base 60 as it relates to time and base 40 because of its application in a future unit. We also build into this a discussion on the history of 0 [Kap99].

The Maya studied astronomy and used three separate calendars: sacred, solar, and historical. The first two are based on regular cycles of length 13 and 20 and the last uses a modified vigesimal system. Calendars are a great place to introduce modular arithmetic. We apply modular arithmetic in calculating future and past days. For example, many stelae reference an event date based on how many days or years after the installation of a ruler or another important occurrence, and so we want to calculate on what date that occurred. One great source of dates in other calendar systems is the Fourmilab website [Swi15]. The students also enjoy making a direct cultural connection by calculating their birth date in the Maya calendar.

Art plays an important role in culture, and is a good topic through which to study symmetry. We discuss symmetry, frieze patterns, and wallpaper patterns and look at examples of textiles to identify the symmetries and patterns. Symmetry also gives us an example of a mathematical group. The students learn the definition of a group and that symmetry and modular arithmetic are examples. This is abstract and so one of the harder topics for students, but we find it worthwhile to give them an introduction to abstract mathematical ideas through the lens of tangible objects found throughout other cultures.

One of the movies we watch in class is *Breaking the Maya Code* [Leb08]. This movie shows the lengthy process and multidisciplinary, collaborative nature of deciphering the Mayan written language. It gives the students a sense of how mathematicians can contribute to other fields of study as well as how pattern finding was important.

Inca

Less is known about the Inca as the accounts we have are colonial and post-colonial. We first learn about the quipu, focusing on quipus which contain numerical data. Quipus are knotted cords that were used to carry information across the vast empire. There are still a lot of open questions about what information quipus contain (just numerical data or also history and stories) and how to read them. Almost no quipus have been tied to written records or been conclusively interpreted. Quipus generally have two organizational structures: cross-categorization and hierarchical. A good resource is the book *Mathematics of the Incas: Code of the Quipu* by mathematician Marcia Ascher and anthropologist Robert Ascher [AA97]. In the course we talk about organizing data in tables as well as learning about tree diagrams. A database of quipus that you can have

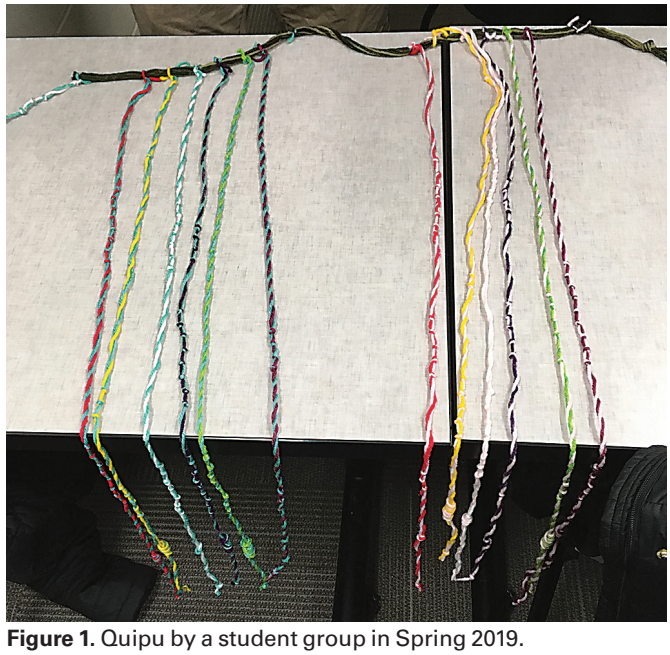


Figure 1. Quipu by a student group in Spring 2019.

students look at (or try to analyze) is the Khipu Database Project [Urt20]. We do not try to fit in some statistical analysis with this topic, but one could. The head of the database project, Gary Urton, also has several books out on the subject. At the end of the unit, students create their own numerical data quipu, and the other students attempt to decipher its meaning.

While it is a loose connection, we also relate quipus to cryptography asking the question of how to break the "code" that the quipu is to us as well as asking how one could repurpose the quipu as a cipher method.

The Inca also used an object called a yupana, which many experts think was used as an abacus. Others think it was used as a game, or perhaps for both purposes. The few colonial mentions of it reference the numbers 1, 2, 3, and 5, and hence we cover the Fibonacci sequence and golden ratio. There are several hypotheses out there on how to use the yupana. We learn and analyze (in terms of mathematics and culture) a few theories, as well as have the students develop their own.

Nasca

The German mathematician Maria Reiche spent most of her career studying the Nazca lines in Peru [Rei49]. Because of the aridity of the desert, these etchings in the sand have been preserved for over 2000 years. The geoglyphs include lines, geometric shapes, and zoomorphic designs. Reiche believed that the glyphs were tied to astronomy; there are other more widely-accepted theories today as researchers have studied them more. We discuss scaling factors and their use in marking distances and angles with a piece of string. We also look at how Reiche used astronomy and what calculations need to be done to see if the lines today line up with the stars around 2000 years ago. Many of the designs and the concept of using lines appear in other Andean cultures, so ethnographic allegory is another tool we discuss. The students end the unit by creating their own designs depicting something relevant in their culture today.

Throughout the topics, we discuss how the needs and interests of a community influence the math and science which develop. For example, agriculture often leads to a study of astronomy which may lead to a calendar or architecture built to line up with celestial events. While we do not delve into astronomy or architecture in our course, those are other good areas to relate to mathematics. Having a vast empire necessitates having a method of communication that can be easily transported, and resources dictate the medium. Consider what conceptual ideas and skill sets you want to impart upon the students, and look for ways they appear in culture.

Figure 2. Nasca-style line drawings by student groups in Spring 2014.

Conclusion

Find something you are passionate about when designing a new course. Also, consider where it will fit into the curriculum in your department or other departments. Collaborate or consult with someone whose primary studies include the culture or topic. Creating a course is very enriching for the professor, but also for the students who get to see a new viewpoint of mathematics. Designing a mathematics course with a focus on culture is an opportunity to teach students that mathematics is more varied than they have seen before. It also is an opportunity to teach an appreciation and respect for indigenous (or other) culture(s). Mathematics is influenced by culture and culture influences the mathematics developed.

Developing this course and other work I have done since then has encouraged me to look at ways to incorporate history and culture into each math class I teach.

References

[AA97] Marcia Ascher and Robert Ascher, *Mathematics of the Incas: Code of the Quipu*, Dover Publications, Inc., Mineola, NY, 1997. Corrected reprint of the 1981 original *Code of the Quipu*. MR1445558

[Coe93] Michael Coe, *The Maya*, Thames and Hudson Ltd., London, 1993.

[Dav98] Nigel Davies, *The ancient kingdoms of Peru*, Penguin, London, 1998.

[Kap99] Robert Kaplan, *The nothing that is: A natural history of zero*, Oxford University Press, New York, 1999. MR1768799

[Leb08] David Lebrun, *Breaking the Maya code*, ARTE and Night Fire Films, 2008.

[Rei49] Maria Reiche, *Mystery on the desert*, Penguin Books, New York, 1949.

[Sal04] Frank Salomon, *Cord keepers: Khipus and cultural life in a Peruvian village*, Duke University Press, Durham, 2004.

[Swi15] Fourmilab Switzerland, Calendar converter, 2015.

[Urt20] Gary Urton, *Khipu database project*, 2020.

Colleen Duffy

Credits

Figures 1 and 2 are courtesy of Colleen Duffy.

Author photo is courtesy of University of Wisconsin–Eau Claire.