PETER G. CASAZZA

1. AN ALGEBRA TEACHER I COULD UNDERSTAND

Emmy award-winning journalist and bestselling author Cokie Roberts once said:

As long as algebra is taught in school, there will be prayer in school.

1.1. An Object of Pride. Mathematician's relationship with the general public most closely resembles "bipolar" disorder - at the same time they admire us and hate us. Almost everyone has had at least one bad experience with mathematics during some part of their education. Get into any taxi and tell the driver you are a mathematician and the response is predictable. First, there is silence while the driver relives his greatest nightmare - taking algebra. Next, you will hear the immortal words: "I was never any good at mathematics." My response is: "I was never any good at being a taxi driver so I went into mathematics." You can learn a lot from taxi drivers if you just don't tell them you are a mathematician. Why get started on the wrong foot?

The mathematician David Mumford put it:

"I am accustomed, as a professional mathematician, to living in a sort of vacuum, surrounded by people who declare with an odd sort of pride that they are mathematically illiterate."

1.2. A Balancing Act. The other most common response we get from the public is: "I can't even balance my checkbook." This reflects the fact that the public thinks that mathematics is basically just adding numbers. They have no idea what we really do. Because of the textbooks they studied, they think that all needed mathematics has already been discovered. They think "research" in mathematics is *library research*. They have no idea that mathematicians can't balance their checkbooks either - although for reasons different from theirs.

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1.3. Accounting to the Public. The public sees us as slightly mad geniuses since we take for granted things they cannot even imagine. They see us as aliens who are just visiting this planet long enough to make their lives miserable. They are not sure if they should be talking to us or running for the exit. They are pleasantly surprised if they discover we can hold a normal conversation with a mere mortal. We like being "geniuses" to the public. If we must have a false mystique, this is probably the best we could ever hope for. The problem is that their definition of "genius" is quite different from ours. That is why they can think of actors as *creative* and mathematicians as *accountants* who can balance checkbooks.

1.4. We Have Major Problems. This view of mathematicians as "geniuses" creates problems. Many students are discouraged from entering mathematics since their teachers don't see them as brilliant enough to be in the field. Believing that success comes from innate talent takes away any control you have over your career. And after being classified as a genius, you try to live up to this expectation only to discover that brilliant inspirations are few and far between. And if you manage to do something spectacular, you can easily become obsessed with trying to outdo this by working only on the *major problems* which have ruined the careers of many mathematicians before you.

1.5. Our "15 Minutes of Fame". We keep waiting for the public to give us some positive form of recognition. But even when they defer to mathematics, it is often facetious such as the common expression today: "You do the math." And it does not help with the public to remove our mantle of *mathematician* and replace it with *faculty* since it just lands us on the sword of Publicist William F. Buckley:

"I would rather entrust the government of the United States to the first 400 people listed in the Boston telephone directory than to the faculty of Harvard University."

Perhaps our relationship with the public is best summarized by the following scene from the television series *Law and Order*. Two police officers are standing over a dead body in a high school classroom.

First officer: "An art teacher. I can't believe she ever hurt anyone."

Second officer: "An algebra teacher I could understand."

2. "All My Imaginary Friends Like Me" : Nikolas Bourbaki

A famous satirist and mathematician Tom Lehrer (Isn't "lehrer" the German word for "teacher"?) once said:

Some of you may have met mathematicians and wondered how they got that way.

2.1. Fulfilling Careers in Mathematics. Mathematicians form a broad spectrum of personalities from "normal" to isolated, introverted, etc. We have a different definition of *normal* precisely so that we can declare ourselves in this category. Mathematicians from a very tender age may see themselves as different. Worse, those around you may see you as different. The word *nerd* arose so people would have at least some definable category to put us into. This partly comes with the territory. The very traits which make us good at mathematics work against us in society. A minimal requirement in mathematics is a certain level of being *obsessive compulsive*. An obsession is a "persistent recurring thought", while a compulsion is "an action a person feels compelled to carry out over and over." What abnormal psychology texts see as a "disorder" - we embrace. They refer to the bad side effects of *obsessive compulsive behavior* as:

"Emphasis on logic and reasoning over feeling and intuition."

"Keeping everything in order and under strict control."

O.K. But I am still waiting for the **bad** side effects?

As if this isn't enough, take a look at *Asperger's Syndrome*. "A condition on the autistic spectrum. It includes repetitive behavior patterns and impairment in social interaction." Finally, they end with the punch line:

"These characteristics can often lead to fulfilling careers in mathematics, engineering and the sciences".

Thank you. We needed that recognition. The main point is that the very traits that make us good as mathematicians - make us not so good at social interactions. So, many mathematicians are quite introverted. Luckily, in a group of mathematicians, you can easily tell the extrovert. She is the one looking at *your* shoes when she is talking to you.

2.2. Adapting to Intelligence. After reading about these "disorders", I am not sure if I should be removing my two favorite signs from my wall:

Gone crazy. Be back shortly.

Anything worth doing is worth overdoing.

People like to say there is a thin line between genius and psychosis. And there are many famous cases where mathematicians fell over the line (Go see

the movie "A beautiful mind". And to get a clearer picture of how "hollywood" sees us make sure you also see: "Proof", "Pi" and "Good WIll Hunting". But remember that this is **fiction** and represents just how others see us.) The problem is that mathematics did not make us "legally" insane, or we could walk away and take care of it. Rather, it was precisely these characteristics which drove us into an area which finds all our strange behaviors as completely normal - even desirable.

Employers like us because we question everything - even those things they have held sacred forever. This gives them a chance of making real needed changes in their company. But these same qualities can alienate those around us who don't like having someone questioning everything. Their worlds are comfortable precisely because they don't constantly question their surroundings.

As if things are not bad enough, it is almost impossible to tell a nonmathematician what we are doing. They don't have the patience or blackboard space to contain the 12 definitions we need to begin the discussion. And if we really try to explain ourselves, we just look even more abnormal to someone who can not comprehend why anyone in this universe - or any parallel universe - could possibly derive excitement from this.

As the mathematician Janet Tremain puts it:

Intelligence is maladaptive.

3. An Explosive Subject

A quote of unknown origin goes:

A clever person solves a problem. A wise person avoids it.

3.1. Entering Nobel's Mind. Alfred Nobel died on December 10th, 1896 leaving the major part of his vast fortune to fund the "Nobel Prize" which was designed to reward "science, literature and the quest for peace." It was to be given to those who "shall have conferred the greatest benefit on mankind." Mathematics was not honored with a prize. For a long time mathematicians speculated about why there was no Nobel Prize in Mathematics. The reasons went from the ridiculous to the sublime. For an area which lives off "truth", it was surprising how easily we were able to distort historical truth. Actually, we have no idea why Nobel did not have a prize in mathematics. I personally believe in the view given by Garding and Hormander: "The true answer to the question [why there is no Nobel Prize in Mathematics] is that, for natural reasons, the thought of a prize in mathematics never entered Nobel's mind."

3.2. A Field of Dreams. At the 1924 International Congress of Mathematicians a resolution was adopted to create medals to recognize outstanding mathematical achievement - later to be called the Fields Medal after J.C. Fields the Secretary of the Congress. The Medal is awarded every four years on the occasion of the ICM to recognize "outstanding mathematical achievement for existing work and for the promise of future achievement." I guess to leave adequate room for "future achievement", the Medal is awarded only to mathematicians below the age of 40. To here all was fine. We have an award for the best of the best young people to recognize their mathematical talents. But this all went awry when mathematicians started referring to this as the mathematical equivalent to the Nobel Prize. This left us looking foolish and ridiculous outside the field when we claimed to have the equivalent to the Nobel Prize but it is not given for the most significant achievements in mathematics, but rather for the *youngest most significant achievements*. Unfortunately, this mistake has worked its way into Wikipedia and a number of other popular venues.

3.3. Exploding With Success. The International Mathematical Union seems incapable of addressing the issue of a Nobel Prize in mathematics. Some people have suggested, for example, using the Abel Prize. The only saving grace here is that a new Nobel Prize in Economics was added in 1969 and so we can hope for our turn one day.

By the way, Alfred Nobel was a Swedish chemist and engineer. A series of disastrous accidents in his lab left a number of people dead (including his youngest brother Emil) while he tried to learn how to stabilize nitroglycerine. Eventually succeeding, he called his discovery **dynamite** and it was this invention which generated the massive fortune he used to fund the Nobel Prize. Nobel himself described 1860 as the time when he "made nitroglycerine **explode with success**."

4. The Goalkeeper

During World War II British Prime Minister Winston Churchill explained how to deal with severe adversity:

If you are going through hell - keep going.

When Jean Bourgain was just starting his career, Janet Tremain (a student at the time) asked him what his goals were. He said with his dry smile (Something which the greatest code breakers of all time could not even begin to decipher): "To win a Fields Medal (The highest level award available at the time); To be at the Advanced Institute (The highest level position in mathematics); and to make a lot of money (Impossible in mathematics)."

4.1. Scoring Goals. When you enter mathematics, you will have to set your goals for your career. If you set your goals too high, you will spend your entire career frustrated and unhappy that you can't achieve what you want. We have enough stories in mathematics already about mathematicians ending their careers bitter and angry that they were not able to live up to their own unrealistic goals. For most of us, all we can hope for is to become "a first rate - second rate mathematician". This is already a lofty goal. There are just a tiny number of first rate mathematicians. Luckily, an army can't move forward if it consists only of generals. It takes a broad spectrum of mathematicians with all kinds of different talents to propel the subject forward. Also, the most critical need in mathematics is for truly creative ideas - and these can come from anyone.

4.2. Today is Yesterday's Tomorrow. Keep in mind also that for most of your career you will be "in progress" working on a project. If you only get enjoyment from major victories, you will have very few moments of happiness. You need to learn to enjoy the *process* of doing mathematics so that you can enjoy every day of your career. Yes, mathematics can be the most frustrating endeavor in the universe. It gives up its riches very grudgingly. And since we have declared this as our profession, we cannot be satisfied without rewards. But we can still enjoy the process of discovery even on a frustrating day when we make *negative progress*. That is a day when we realize that what we thought we did yesterday is false. But we can't move forward without that small insight. When asked once how his day was going, the cartoon figure Charlie Brown responded:

I keep hoping yesterday will be better.

4.3. Moving the Goalposts. Mathematics can be the perfect partner - fulfilling your every need. Or it can be your greatest nightmare - frustrating you at every turn. You are the *goalkeeper* of your career. If you tie your ego to your success, you will end up as a ring without identity.

By the way, Bourgain reached all three of his "goals" and much more (such as becoming a member of the National Academy of Sciences) at a very young age. So be sure not to set your goals too low. Or at least be prepared to move the goalposts as you go along.

5. Group Theory

Since your career in mathematics might be lengthy, please keep in mind the fundamental rule for long term group interaction:

Friends come and go, but enemies accumulate.

5.1. Simple Groups. When I went to my first mathematics meetings I noticed that the "stars" all hung out together, ate together etc. I assumed that we were not to disturb *the group* until we had built up enough *mathematical currency*. I later learned that this was not necessarily the case. Over years of being in a subject, other mathematicians may be our longest lasting friends. We have shared their marriages, raising of children (bunnies in my case) and the ups and downs of life. So we are glad to see old friends and catch up on what is going on with them. This is not designed as a slight on all the people we don't know so well. Getting into the group may not be a completely simple matter. But in my groups, at least, it is semi-simple. So give it a try.

5.2. The Inverse Elements. If things don't work out as you expected, don't let this discourage you. At my first math meeting I stopped Mr. Big in the hallway and said: "Hi. I am Pete Casazza. Would you have time to answer a question?" The response was: "Who are you?" Since I had a name tag and had just introduced myself, I quickly realized that this was **mathematics speak** for "Why are you important enough for me to talk to?" Since it was my first meeting, I concluded that overrating one's importance in the universe was a necessary condition for being a mathematician. Actually, such an experience is rare in mathematics and nothing near it ever happened to me again. But be prepared to meet a very broad spectrum of egos during your career.

6. MATHEMATICS IS AGELESS

The comedian Jack Benny once said:

Age is a question of mind over matter. If you don't mind, it doesn't matter.

6.1. The Age of Reason. Mathematicians have a paranoia about age. We are told from our earliest days that mathematics is a young person's game; that mathematicians do all their best work before the age of 40. This paranoia is made worse by the fact that the Fields Medal (see Section 3) has to be won before the age of forty. For a group which lives off logic, it is difficult to see how they so desperately hold onto this idea despite being faced with a very large number of counter-examples amongst us. Under this system, we should hand someone their Fields Medal for being the best and the brightest of the younger generation and say: "Here is your medal. By the way, your career is over."

6.2. Our Silver Anniversary. At one time, we would hold a special conference to honor a mathematician's 70th birthday. This made sense because that was often the forced retirement age. Today we have broadened our scope to honor the 60th, 65th, and 70th birthdays. But there is a segment of the

community who is afraid to have this "honor" since it is tantamount to announcing the end of your career. If this age paranoia continues then I would suggest we stop having such age related honor meetings. Instead, why not have a special meeting for the 25th (30th etc.) anniversary of someone's entering mathematics? This would, of course, be your *silver* anniversary. But it would be better if we could just face this whole topic realistically in the first place.

7. WORKMAN'S COMPENSATION

A famous actor Will Smith claimed that he had only average talent. That all his success stemmed from hard work.

"When the other guy is sleeping, I am working. When the other guy is vacationing, I am working. When the other guy is making love, I ... uh ... well, I am doing the same. But I am working really hard at it."

7.1. Input verses Output. As a student thinking about entering mathematics, don't be intimidated by its mystique. You do not have to be a genius to be a mathematician with a successful career. It helps to be of above average intelligence, but the most important tool at your disposal is *hard work*. Mathematics is not a *sprint* but rather a *marathon*. Hard work over a long period of time will pay off. It has been noted that today a large percentage of all gifted students severely underestimate their abilities. This comes from underrating the importance of effort. The system designed to help them is actually working against them. They are praised for being "gifted" which is something out of their control and therefore has limitations. When praised for their hard work instead, gifted students usually set higher standards, take more risks to succeed and expect more of themselves. As Tom Lehrer once said:

Life is like a sewer. What you get out of it depends upon what you put into it.

If you put your best efforts into mathematics you have a reasonably good chance of a successful career.

7.2. Some Inspiration. Fields Medalist Terence Tao put work in its place quite eloquently:

"The popular image of the lone (and possibly slightly mad) genius - who ignores the literature and other conventional wisdom and manages by some inexplicable inspiration (enhanced, perhaps, with a liberal dash of suffering) to come up with a breathtakingly original solution to a problem that confounded all the experts - is a charming and romantic image, but also a wildly inaccurate one, at least in the world of modern mathematics. We do have spectacular, deep and remarkable results and insights in this subject, of course, but they are the hard-won and cumulative achievement of years, decades, or even centuries of steady work and progress of many good and great mathematicians."

"Actually, I find the reality of mathematical research today - in which progress is obtained naturally and cumulatively as a consequence of hard work, directed by intuition, literature, and a bit of luck - to be far more satisfying than the romantic image that I had as a student of mathematics being advanced primarily by the mystic inspirations of some rare breed of "geniuses".

We have an expression for this in mathematics (which is based on a quote of Edison):

Success in mathematics is 1% inspiration and 99% perspiration.

8. A CONFIDENCE GAME

When Nobel Prize winning Physicist Albert Einstein's father asked the school principal what vocation his son should choose, the response was:

It doesn't matter, he'll never succeed at anything.

8.1. The Advantages of Mathematics. One of the greatest challenges to your career will be to maintain your confidence - without being over-confident. Everything around you will be constantly testing your confidence. You are working on problems which appear unsolvable. You are constantly being evaluated for grants, jobs, promotion, tenure, raises etc. Deciding who will be the main speakers at meetings will be an evaluation process. Even when you achieve a great victory in your research, the first question that arises is: "Can I top this?" No matter how good you are, there is *always* someone better out there. There will be people around you who are faster, more knowledgeable, and more creative than you. This does not mean that you don't belong in mathematics. As T. Tao put it:

"This is the common mistake of mistaking **absolute advantage** for **comparative advantage**. ... As long as you have education, interest, and a reasonable amount of talent, there will be some part of mathematics where you can make a solid and useful contribution."

8.2. In Praise of Mathematics. You will have to maintain your own confidence. Mathematicians are very stingy with praise. You will likely never hear someone say: "That was a great theorem. Thank you for bringing it to us."

Or "Your book has greatly improved my mathematical life." Unfortunately, this is not part of the psyche of mathematicians - but it should be. For some *[natural?]* reasons, the thought of praise never enters mathematician's minds.

8.3. True or False Questions. One of the most difficult problems in mathematics is learning how to balance *proper* respect for our *significant others* while maintaining a *healthy* respect for ourselves. If your whole measure of a human being is their mathematical achievements, you will be constantly undermining your own confidence. I often admonish my students for talking to themselves in the negative: "That was a dumb statement" or "That was really stupid on my part". I believe they became accustomed to doing that as a defense mechanism. If they say it first, it takes away the opportunity for others to say it. But every psychological study around shows that how we *talk to ourselves* is being heavily recorded in our subconscious and is forming our view of ourselves. Mathematical statements are not *smart or dumb*. They are only *true or false*.

You cannot afford to lose the **Confidence Game**: "A swindle in which the victim is defrauded after his or her confidence has been won."

9. You Can't Outrun a Bear

The comedian George Carlin once said:

Where are we going? And what's with this hand basket?

9.1. Wonderful Advances. When I first joined the mathematics community I was excited to join a group dedicated to *advancing mathematics*. I had a rude awakening when it became clear that we were really working to *advance ourselves*. This is an unfortunate consequence of the reality around us. We must all compete for very scarce research grants, positions, promotion, tenure, awards, raises etc. But we need to be careful that this reality does not diminish our enjoyment of the subject. We need to be able to go to meetings and be excited and overjoyed at some wonderful advances - *done by someone else*. One always goes away feeling a little behind, but this is one of the main functions of meetings. They infuse us with added energy and drive to go home and do something serious.

9.2. A "Non-Profit" Organization. Mathematics is going through some difficult times at the moment. We have a shortage of jobs, too low salaries, much of our funding is disappearing as the defense department shifts funds from mathematics and the National Science Foundation is funding only 10%-20% of the submitted proposals. Even NSF's figure is quite understated since more and more mathematicians are not even applying to NSF because the chances of being funded are so slim. At this time, the NSF budget is about \$6

billion - approximately what the government will spend in 14 hours. Apparently, no one told them that serious researchers put in a 28 hour work day. An added problem is that even if NSF gets an "inflationary raise", the research they can support diminishes since they are supporting the most significant researchers in the sciences and engineering who are getting significantly above an inflationary raise. Apparently, the U.S. Government sees us as a *not for profit* group. They seem to have forgotten that this country has reached its current status by being the world leader in research. And while they consider increasing research funding to deal with the monumental problems facing the U.S. and the world, they are clueless that these problems would not even be here if they had adequately funded research in the first place. Worse, as they *target* research money to what *they* believe will give immediate relief to our problems, they will continue to underfund the futuristic research which represents the real long term future of the country - leaving us to wait for the next crisis we are not prepared for. This brings to mind an old proverb:

If two wrongs don't make a right - try a third.

9.3. Assisted Suicide. This is the right time for us all to come together for the good of the subject. We have three major math societies in the U.S.: The Mathematical Association of America (MAA) representing students and teachers of mathematics; The American Mathematical Society (AMS) representing pure mathematics; and the Society for Industrial and Applied Mathematics (SIAM) representing *applied* mathematics. Since I worked for 25 years in pure math and then switched into applied math and have been an active participant in the MAA, I have had an opportunity to witness all these groups in action. The pure math group looks down on the applied math group claiming they are not doing *serious* math. The applied math group looks down on the pure math group claiming they are developing deeper and more isolated theories which not only separate themselves further from applications but are isolating themselves even from other areas of pure math. Both of these groups have a certain level of disrespect for the "teaching wing". This is somewhat ridiculous since most of us are employed as teachers and doing research is desirable (or even required) but not viewed as our most important function by state legislatures.

At one time, all of mathematics grew out of applications. To establish its identity, it was natural that mathematics would separate itself from applications to build an independent future. But now it is time for us to come together. When I switched into applied math, the first thing I discovered was that some of the most important questions in pure math were not being addressed because they did not show up naturally there. They only showed up when one tried to apply the theory. Given the significant challenges facing

mathematics, it is time for all the societies and mathematicians of all persuasions to come together for the good of the subject. But our natural inclination seems to be to compete instead of cooperate. Mathematics has become its own worst enemy - assisting in our own suicide.

9.4. **Primitive Ideals.** To understand all of the *apparent* contradictions above, I need to recall a story we used to tell in the boy scouts.

Two boy scouts are out hiking when they see a bear charging at them. One scout sits down, takes his running shoes out of his backpack and starts to put them on. The conversation goes:

First Scout: Why are you putting on running shoes? You can't outrun a bear.

Second Scout: I don't have to outrun the bear. I only have to outrun you.

10. A Black Cat Which Isn't There

A quote sometimes credited to Einstein goes:

"If I knew what I was doing, it wouldn't be called research."

Mathematics differs from the other sciences in that we are attempting to *capture truth* while in other sciences they are trying to *approximate truth*. We also differ from the other sciences in that new discoveries don't falsify the old ones but instead extend what was known to capture a *broader truth*. What changes over time is our understanding of the mathematics - what it means and how it fits into the broader picture.

A story in the mathematics community is used to explain the difference between a mathematician and a physicist. Physicists work for ten years on a difficult problem and when they are done they say: "I am a genius for figuring this out." Mathematicians work for ten years on a problem and when they are done they say: "I am an idiot. The answer was obvious." Although this is exaggerated - as this whole article is - it does contain a shred of truth. Most mathematics, once completely uncovered seems somewhat obvious. But this should not be used to downgrade the enormous effort that went into it the first time.

As Charles Darwin put it:

A mathematician is a blind man in a dark room looking for a black cat which isn't there

11. My Most Read Paper

A famous review of a published math paper reads:

The results in this paper are false. The mistakes are not new.

11.1. Name Dropping. When you enter mathematics, you hope to produce that significant manuscript which will identify you forever as a major player in the field. We actually associate many mathematicians with their most significant contributions. Andrew Wiles will forever be attached to his solution to the 300 year old problem affectionately known as: "Fermat's Last Theorem". But this system also has its drawbacks. Max Zorn will forever be revered in mathematics for his *discovery* of Zorn's Lemma. Unfortunately, he made this discovery in his PhD thesis and our association negates his whole illustrious career after that. But the fact is that few mathematicians will ever reach that level of significance and recognition. You will need to be satisfied with a name tag at meetings.

11.2. The Citation Index. You will have to develop enough confidence in yourself to be comfortable around people who just assume they are smarter than you. Otherwise, you will constantly be trying to broadcast your achievements to build up your own confidence. In my department, just in case you missed the significance of someone, they feel compelled to remind you with a complete lack of subtlety: "In my recent paper in the *Annals*, I showed ...". This is **mathematics speak** for: "I am important since my last paper appeared in the highest level pure math journal." Not to be outdone, I cannot resist tooting my own horn and telling them about:

My Most Read Paper

One time I received an urgent message from a journal saying that I was holding up publication since I had not returned the galley proofs of my article. I replied that I had never received them. They resent them overnight mail and I managed to get them back quickly. Two months later, the original galley proofs arrived in a completely mangled package with multiple wrappings, tape and string. Inside was a disk which contained the tex file of my article. On the various levels of covers it was clear that my disk had racked up a large number of frequent flyer miles. Its first trip was to Columbia, South America. Apparently, someone at the post office decided that Columiba, MO was in South America. The next stop for the manuscript was Venezuela followed later by Argentina. The last two addresses were in Washington, D.C. It isn't hard to figure out what happened. This strange disk with all its tex symbols and large number of \$dollar signs\$ ended up in the country of Columbia where

they quickly realized that this was something of real significance. Clearly these strange symbols and the very large number of dollar signs represented the entire operation for the largest drug cartel in the area. All they had to do was put enough agents on it to decipher it. Failing this, they enlisted the help of the secret service in Venezuela who certainly would decipher it. After passing through Argentina, there was only one group left who had a chance of breaking up this drug ring and that had to be in Washington, D.C. By my estimate, untold hundreds of dedicated law enforcement officials went through this manuscript with a fine tooth comb. This certainly has to be my most read manuscript - even if it is a little short of citations.

12. The Real Beauty of Mathematics

Author unknown:

The difference between genius and stupidity is that genius has its limits.

12.1. An Intelligence Test. Many mathematicians think of themselves as being more intelligent than the general public and even other scientists. Such a conclusion requires one to change the definition of IQ which is basically just that which intelligence tests measure. Intelligence tests do not even measure any non-trivial mathematics. They do measure elementary logical reasoning which most scientists use but only mathematicians have turned into a god. But there is no evidence supporting our innate belief in our superior intelligence. Certainly, one needs a certain amount of intelligence to work in mathematics. But we feel compelled to define *intelligence* as *being good at mathematics.* This is what allows faculty to sit in the lounge talking about how stupid the students are. In whose dictionary is "not being good at mathematics" the definition of stupid? That is, one does not have to be good at mathematics to be intelligent. The problem is that when one devotes their whole lives to a subject, they naturally begin to believe this is the only important thing to do. Grothendieck was one of the greatest minds of the last century. His manuscripts kept mathematicians busy for forty years just trying to fully understand this oracle. One day Grothendieck walked away from mathematics and became a farmer. This sacrilege was an unending topic of conversation forever at meetings since mathematicians just could not comprehend that someone so good at mathematics would choose not to do it. Clearly, if you are brilliant at math, you *must* do it.

12.2. **Real Talent.** It is difficult to work in mathematics without developing a certain amount of arrogance. Arrogance stems from seeing ourselves as better, smarter and more intelligent than others - both inside and outside of

mathematics. But this is not an winning long term strategy. It only works as long as *others* are the idiots you think they are. But when someone significantly better than you comes along [and they always do come along in mathematics] you will spend the rest of your career trying to raise arrogance to the level of an art form so it will hide your clinical depression. There is a long list of famous mathematicians who ended their careers bitter and unfulfilled. It is easy to be arrogant. But it takes real confidence, intelligence and talent not to be. Arrogance will not prevent you from being a significant mathematician, but it will remove 50% of the rewards - which involve your interactions with a spectacular group of dedicated mathematicians.

Perhaps the mathematician Janet Tremain summed it up best:

The real beauty of mathematics

is that

you don't have to be intelligent to do it.

13. An Eating Disorder

Professional baseball player Yogi Berra was asked by a waiter if he wanted his pizza cut into four pieces or eight pieces. He replied:

Four. I don't think I can eat eight.

Going to dinner with mathematicians is a culinary experience from the *fourth dimension*. Only mathematicians could turn dinner into such a level of competition that it would qualify as an *Olympic event*. It is a four hour ordeal which *sometimes* even includes 30 minutes of actual eating.

13.1. We've Got to Lower Our Standards. Our first job is to pick the restaurant. This is where wisdom and experience pays off. Having spent enough meals denying that I had anything to do with the selection of this "god-awful place", I play *silence of the lambs.* As the experts work out the definition of a good restaurant, the rest of us are recalling the last time we interfered and thought we might have heard: "When I want your opinion - I will give it to you." After 30 minutes, we have finally arrived at a consensus - that we are hopelessly deadlocked and getting very hungry. We decide that our only choice is to walk down the street and see what is available. After walking far enough for the person behind me to work out a new *stopping time algorithm* (I wish I had put on my running shoes) and reading enough menus to fill the entire *conference proceedings*, we have come to agreement on a place. Unfortunately for us, by the time we all read the menu, the place closes. We realize that we have no choice but to *lower our standards* while a

few restaurants are still open. We decide to take the next open place, whatever it is.

13.2. A Seat at the Table. Entering Joe's: Eat At Your Own Risk restaurant, we proceed to the ancient ritual of *correct* seating order - which is determined by one's *significance* to the mathematics community. Basically, this amounts to organizing an elephant stampede into the order of importance of the elephants. This can make *musical chairs* look like child's play. There are more than 87 billion different ways 14 people can sit around a table and these guys are determined to try every one. Unfortunately, allowing 1 minute to try out each new position, it would take more than 160,000 years to exhaust all possible ways of seating 14 people around a table. It is best if you just sit at the bar and discuss algebra with the bartender until the dust settles. Anyway, you are at least guaranteed a seat this way.

13.3. Stop Global Whining. Next is the wine tasting competition. This has several different events such as: Who knows the best wines? Who knows the most expensive wines? Who tasted the most wines on the list - and where? Who drinks only wines from their own country - and why? Some people are already begging me for the glass of wine I brought from the bar. Finally we get through the last part of the competition: Who can send back the most bottles of wine? It is over except for assigning grades to the winners: It is too young, too warm, too cool, too spicy, doesn't go with the meal ... I give my two cents: At least the cork tastes good and the label is pretty.

13.4. Extreme Sports. Dinner discussion for mathematicians raises competition to a higher plane. We are compelled to establish some sort of mathematical pecking order: the best, the worst, most creative, most potential ... Then we need to relive the same dimensions with side conditions: biggest drinker, oldest, youngest, most arrogant, best driver, knows a lot about ... (Did I just hear the name Grothendieck mentioned?). We feel compelled to identify all the extremes. After all, being average at anything is the greatest insult we can assign. Didn't we have this discussion last year? Who won?

13.5. **A Taxing Situation.** Finally, the waiter arrives and asks: "Do you want separate checks?" No thanks, we can do the math. Everyone starts talking amongst themselves computing what and how much or anything shared they had. We wind our way through bartering, trading, assigning weights for students, post-docs, untenured professors (why not emeritus?) grant supported or not, tax/tips etc. Running out of napkins to write on, someone is using the tablecloth as a calculator while others stare at the ceiling or their own feet trying to concentrate on the calculations. Each of us finally gets our

bill computed to the exact cent. Who says we can't balance a checkbook? Unfortunately, everyone has only \$20 bills. It is time to start making out the I.O.U.'s. Does anyone have any paper?

As we get up from the table to leave, I make my first serious mistake of the evening (there goes my "no-hitter") and ask: "Does anyone know the best way to get home?" . . . God I wish I never said that. There's 30 minutes of my life I will never get back.

14. The Introduction - Finally

This article is intended as a survival guide for those students, teachers and mathematicians who are having trouble interpreting the mathematical experience. If you read only this article, you will get a distorted image of the overall situation in mathematics. A comprehensive view of the subject would fill a textbook - preferably one on abnormal psychology. In lieu of this, the MAA has put out this book. Therefore, this article is purposely representative of nothing but my own personal experiences during 36 years of being a mathematician. To get a better view of mathematics and mathematicians, you will need to read the other carefully crafted articles in this book. I strongly recommend you visit Terence Tao's web page for more wisdom on these topics than any mathematician should ever be allowed to possess. Each issue raised in this article needed much more discussion. But my goal was only to raise these topics as items that the mathematics community needs to address. My long list of personal opinions on them is not particularly important. It was just important that I make them controversial enough to stimulate the much needed discussion.

If my article annoys you so much that you feel compelled to speak out – please do so since it means that I have done my job. This book is not designed to be the beginning of the end of the story but rather the end of the beginning. My satirical approach to this task is my response to the *sad fact* that no one has ever accused mathematicians of not taking themselves seriously enough.

15. The Last 10 Minutes

Being a mathematician is the greatest job in the world. Every day is even more exciting than the last. Each day starts with mental gymnastics. And today has the potential of being the day that you finally crack that tough nut. It is a challenging, very stimulating, treasure hunt. It is creative brainstorming at its best. Most of the non-mathematicians my age I know are already burned out and are trying to hang on until retirement. But even after 36 years of doing mathematics, I go to bed at 2 pm in the afternoon and get up at 10 o'clock at night so I can get to the thing I love as early as possible. This of course occurred because of my obsessive compulsive nature. I started getting up at

5 am so I could get two hours of research done before going to the university. Then I figured out that getting up at 4 am gave me an extra hour. After a short time I was getting up at 10 pm. Pretty soon I won't have to go to bed since I will be getting up at that time. The point is, few people will have the opportunity to have a career that is so constantly exciting, rewarding and challenging that they cannot imagine retiring from it. When you get into it you will realize that a day without mathematics is a day without sunshine. Or as a famous expression goes:

Mathematics is not a matter of life and death.

It is much more important than that.

So if you are not yet one of us, come join in. Lighten up, chill out, relax ... it's just mathematics and you are just a person. Enjoy the treasure hunt.

Someone asked me once if I planned on doing mathematics my whole life. I gave the obvious answer:

"Of course not. I plan on saving the last 10 minutes to reminisce."

Acknowledgment I fretted for almost a year over how I was going to write (or even approach) this article. Then I attended the GPOTS conference at the University of Cincinnati and spent the week carefully discussing this article with my friends and colleagues. It soon was completely clear what I needed to write. Returning home, I wrote the first draft of this article in two days. I am indebted to everyone who attended the meeting for all their insights and for just being their wonderful selves. A special thanks goes to Don Hadwin for supplying the creative title for Section 2. Many others sent much needed improvements: Karlheinz Gröchenig, Chris Heil, Norbert Kaiblinger, Jelena Kovačević, Sergei Novikov. A large portion of the material for this article came from Janet Tremain's eidetic memory. I sat for hours while she played video tapes stored in her mind recounting minute details of events in our past interactions with mathematicians. One example: she replayed a dinner we had twenty years ago with a group of mathematicians, describing where each person sat at the table, what each person ate, who sent back their steak to be cooked more, what wine/beer each person drank, and each conversation going on at the table. My response: "Janet, you are scaring me!"

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