TEN SIMPLE RULES
FOR
MATHEMATICAL WRITING

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ON WRITING

- "Easy reading is damn hard writing" (Hawthorne)

- "Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought" (Knuth)

- "I think I can tell someone how to write but I can't think who would want to listen" (Halmos)

WHAT IS MATH WRITING?

- **Writing where mathematics is used as a primary means for expression, deduction, or problem solving.**

- **Examples that are:**
  - **Math papers and textbooks**
  - **Analysis of mathematical models in engineering, physics, economics, finance, etc**

- **Examples that are not:**
  - **Novels, essays, letters, etc**
  - **Experimental/nonmathematical papers and reports**

WHAT IS DIFFERENT ABOUT MATH WRITING?

- **Math writing blends two languages (natural and math)**
  - **Natural language is rich and allows for ambiguity**
  - **Math language is concise and must be unambiguous**

- **Math writing requires slow reading**
  - **Often expresses complex ideas**
  - **Often must be read and pondered several times**
  - **Often is used as reference**
  - **Usually must be read selectively and in pieces**

WHY THIS TALK?

- **Experience is something you get only after you need it …**

- **One current model:** **The conversational style**
  - **"Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods" (Halmos)**
  - **"Talk to your readers as you write" (Strang)**
  - **Very hard to teach to others ("Effective exposition is not a teachable art. There is no useful recipe …" Halmos)**
  - **Controversial (where do proofs start and end? … I am not sure what the assumptions are … I can't find what I need … etc)**

- **Instead we will advocate a structured style**
  - **Offers specific verifiable rules that students can follow and thesis advisors can check**
  - **Allows room to develop and improve over time**

SOURCES

- **General style books**
  - Strunk and White, "The Elements of Style" (www)
  - Fowler and Aaron, "The Little Brown Handbook"
  - Venolia, "Write Right!"

- **Halmos, "How to Write Mathematics"**

- **Knuth, et al, "Mathematical Writing" (www)**

- **Kleiman, "Writing a Math Phase Two Paper," MIT (www)**

- **Krantz, "A Primer of Mathematical Writing"**

- **Higham, "Handbook of Writing for the Mathematical Sciences"**

- **Alley, "The Craft of Scientific Writing"**

- **Thomson, "A Guide for the Young Economist"**

RULES OF THE GAME

- **Small rules:**
  - Apply to a **single sentence** (e.g., sentence structure rules, mathspeak rules, comma rules, etc)

- **Broad rules:**
  - Apply to the entire document
  - **General** style and writing strategy rules
  - Are **non-verifiable** (e.g., organize, be clear and concise, etc)

- **Composition rules (our focus in this talk):**
  - Relate to how parts of the document connect
  - Apply to **multiple sentences**
  - Are **verifiable**

SOME EXAMPLES OF SMALL RULES I

- **Break up long blocks of text into simpler ones**
  - **2-3-4 rule:** Consider splitting every sentence of more than 2 lines, every sentence with more than 3 verbs, and every paragraph with more than 4 "long" sentences.

- **Mathspeak should be "readable"**
  - **BAD: Let k>0 be an integer.**
  - **GOOD: Let k be a positive integer** or **Consider an integer k>0.**
  - **BAD: Let $x \in R^n$ be a vector.**
  - **GOOD: Let x be a vector in R^n** or **Consider a vector $x \in R^n$.**

- **Don't start a sentence with mathspeak**
  - **BAD: Proposition: f is continuous.**
  - **GOOD: Proposition: The function f is continuous.**

SOME EXAMPLES OF SMALL RULES II

- Use active voice ("we" is better than "one")

- Minimize "strange" symbols within text

- Make proper use of "very," "trivial," "easy," "nice," "fundamental," etc

- Use abbreviations correctly (e.g., cf., i.e., etc.)

- Comma rules

- "Which" and "that" rules

- … ETC

SOME EXAMPLES OF BROAD RULES

- **Language rules/goals to strive for: precision, clarity, familiarity, forthrightness, conciseness, fluidity, rhythm**

- **Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)**

- **"Down with the irrelevant and the trivial" (Halmos)**

- **"Honesty is the best policy" (Halmos)**

- **"Defend your style" (against copyeditors - Halmos)**

- **... ETC**

THE TEN COMPOSITION RULES

- **Structure rules (break it into digestible pieces)**
  - **Organize in segments**
  - **Write segments linearly**
  - **Consider a hierarchical development**

- **Consistency rules (be boring creatively)**
  - **Use consistent notation and nomenclature**
  - **State results consistently**
  - **Don't underexplain - don't overexplain**

- **Readability rules (make it easy for the reader)**
  - **Tell them what you'll tell them**
  - **Use suggestive references**
  - **Consider examples and counterexamples**
  - **Use visualization when possible**

1. ORGANIZE IN SEGMENTS

- **"Composition is the strongest way of seeing" (Weston)**

- **Extended forms of composition have a fundamental unit:**
  - **Novel**   **Paragraph**
  - **Film**   **Scene**
  - **Slide presentation**   **Slide**
  - **Evening news program**   **News report**

- **Key Question: What is the fundamental unit of composition in math documents?**

- **Answer: A segment, i.e., an entity intended to be read comfortably from beginning to end**

- **Must be not too long to be tiring, not too short to lack content and unity**

SEGMENTATION PROCESS

- **Examples of segments:**
  - A mathematical result and its proof
  - An example
  - Several related results/examples with discussion
  - An appendix
  - A long abstract
  - A conclusions section

- **A segment should "stand alone" (identifiable start and end, transition material)**

- **Length: 1/2 page to 2-3 pages**

SEGMENT STRUCTURE

Title (optional)

↓

Transition Material

↓

Definitions, Examples
Arguments, Illustrations

↓

Transition Material

EXAMPLE OF SEGMENTATION: A SECTION ON PROB. MODELS

- **Sample space - Events** (1 page)
- **Choosing a sample space** (0.5 page)
- **Sequential models** (0.75 page)
- **Probability laws - Axioms** (1.25 page)
- **Discrete models** (2 pages)
- **Continuous models** (1 page)
- **Properties of probability laws** (2 pages)
- **Models and reality** (1.25 page)
- **History of probability** (1 page)

See Sec. 1.2 of Bertsekas and Tsitsiklis probability book

2. WRITE SEGMENTS LINEARLY

- Question: What is a good way to order the flow of deduction and dependency?

- General rule: Arguments should be placed close to where they are used (minimize thinking strain)

- Similarly, definitions, lemmas, etc, should be placed close to where they are used

- View ordering as an optimization problem

- A linear/optimal order is one that positions arguments (definitions, lemmas) so as to minimize the total number of "crossings" over other arguments (definitions, lemmas), subject to the dependency constraints. Depth-first order is usually better.

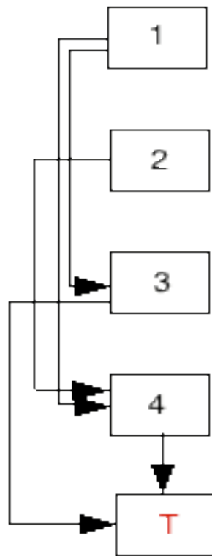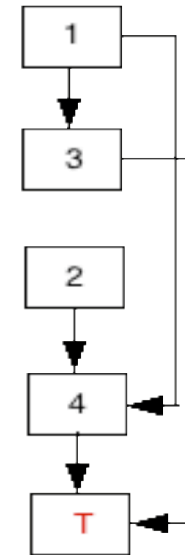# EXAMPLES OF ORDERING

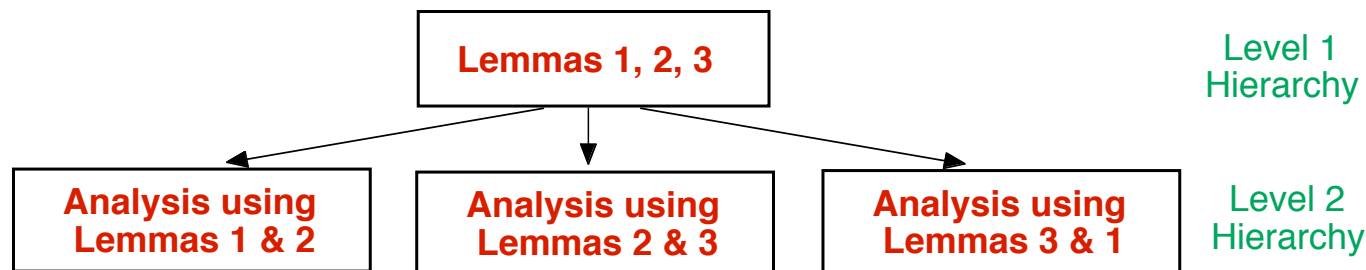Dependency Graph of Arguments
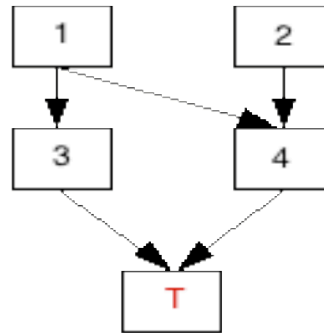
Level 1 Arguments

Level 2 Arguments

Nonlinear

Linear

3. CONSIDER A HIERARCHICAL DEVELOPMENT

- **Arguments/results used repeatedly may be placed in special segments for efficiency**

Lemmas 1, 2, 3 — Level 1 Hierarchy

Analysis using Lemmas 1 & 2 | Analysis using Lemmas 2 & 3 | Analysis using Lemmas 3 & 1 — Level 2 Hierarchy

- **Possibly create special segments for special material (e.g., math background, notation, etc)**

- **Analogy to subroutines in computer programs**

4. USE CONSISTENT NOTATION

- **Choose a notational style and stick with it**

- **Examples:**
  - **Use capitals for random variables, lower case for values**
  - **Use subscripts for sequences, superscripts for components**

- **Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc**

- **Use simple notation. Example: Try to avoid parenthesized indexes: x(m,n) vs x_{mn}**

- **Avoid unnecessary notation:**
  - **BAD: Let X be a compact subset of a space Y. If f is a continuous real-valued function over X, it attains a minimum over X.**
  - **GOOD: A continuous real-valued function attains a minimum over a compact set.**

5. STATE RESULTS CONSISTENTLY

- **Keep your language/format simple and consistent (even boring)**

- **Keep distractions to a minimum; make the interesting content stand out**

- **Use similar format in similar situations**

- **Bad example:**
  - **Proposition 1: If A and B hold, then C and D hold.**
  - **Proposition 2: C' and D' hold, assuming that A' and B' are true.**

- **Good example:**
  - **Proposition 1: If A and B hold, then C and D hold.**
  - **Proposition 2: If A' and B' hold, then C' and D' hold.**

6. DON'T OVEREXPLAIN - DON'T UNDEREXPLAIN

- **Choose a target audience level of expertise/background (e.g., undergraduate, 1st year graduate, research specialist, etc)**

- **Aim your math to that level; don't go much over or under**

- **Explain potentially unfamiliar material in separate segment(s)**

- **Consider the use of appendixes for background or difficult/specialized material**

7. TELL THEM WHAT YOU'LL TELL THEM

- **Keep the reader informed** about where you are and where you are going

- Start each segment with a short introduction and perhaps a road map

- Don't string together seemingly aimless statements and surprise the reader with "we have thus proved so and so"

- Announce your intentions/results, e.g., "It turns out that so-and-so is true. To see this, note …"

- Tell them what you told them

8. USE SUGGESTIVE REFERENCES

- **Frequent numbered equation/proposition referencing is a cardinal sin**

- **It causes page flipping, wastes the reader's time, and breaks concentration**

- **Refer to equations/results/assumptions by content/name (in addition to number), e.g., Bellman's equation, weak duality theorem, etc**

- **Repeat simple math expressions**

- **Remind the reader of unusual notation, and earlier analysis**

- **Dare to be repetitive (but don't overdo it)**

9. CONSIDER EXAMPLES AND COUNTEREXAMPLES

- **"Even a simple example will get three-quarters of an idea across" (Ullman)**

- **Examples should have some spark, i.e., aim at something the reader may have missed**

- **Illustrate definitions/results with examples that clarify the boundaries of applicability**

- **Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions**

10. USE VISUALIZATION WHEN POSSIBLE

- "A picture is worth a thousand words"

- Keep figures simple and uncluttered

- Use substantial captions

- Captions should reinforce and augment the text, not repeat it

- Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality

- Prefer graphs over tables

THE END

"**Bad thinking never produces good writing**"

(Lamport)

Good writing promotes good thinking …