Lobachevsky to Luzin: The Trials and Tribulations of Abstract Mathematics in early 20th Century Russia

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Russian Prominence in Mathematics

Three mathematical powerhouses in the 19th and early 20th centuries: Russia, France, Germany.

Different Styles:

- French mathematics often focused on generality and formalism, even at the cost of clarity or readability. (Example: Bourbaki)
- Germans start from clear principles, then make progress step by step, patiently, painstakingly, at a pace following the rules of deductive logic with discipline with extreme severity.
- The Russian style of mathematics tends to focus on the essence rather than the formalism, and emphasize what is novel. It is simultaneously rigorous and accessible.

Russian Prominence in Mathematics

The Soviets built an educational system that, for all of the horrors and inhumanity of its surrounding totalitarian system, produced a striking record of huge successes and rare achievements.

A few words on Russian Math Pedagogy and Math Circles.

One of the best traditions of the Russian school is the unity of research and teaching.

Axiomatic Systems

Euclid and the ancient Greeks

Axiomatic System: Begin with axioms that are assumed to be true, use logic to deduce theorems (true statements) via proofs.

Euclid's Five Postulates for Plane Geometry

1. There is a straight line between any two points.
2. Any line segment may be extended indefinitely along a straight line.
3. For every point and every length, there exists a circle centered at that point with radius equal to the given distance.
4. All right angles are equal to one another.
5. Given a line and a point not on it, there is *exactly one* line parallel to the given line through the point.

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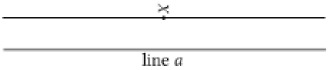

Euclidean Space

Lobachevsky and Non-Euclidean Geometry

Nikolai Lobachevsky, 1792–1856

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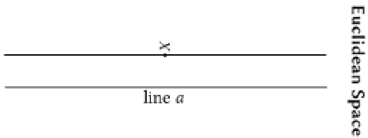
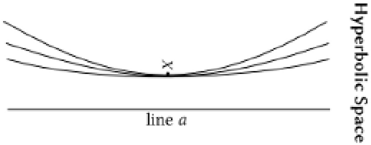
New Axiom: Given a line and a point not on it, there is *more than one* line parallel to the given line through the point.

This led to a new geometry: Lobachevskian (or Hyperbolic) Geometry.





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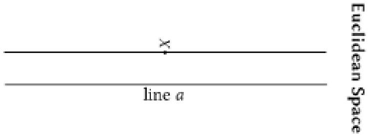
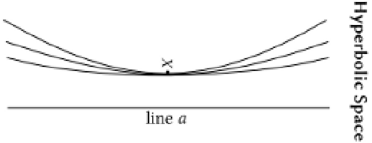
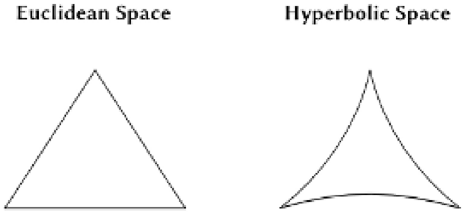




New Theorems:



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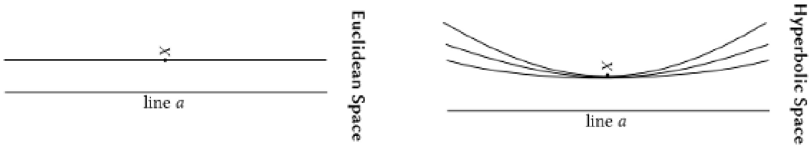

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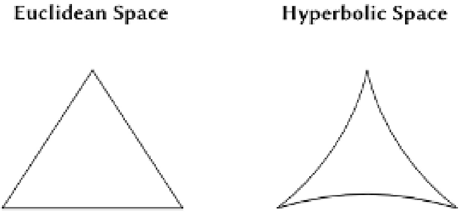
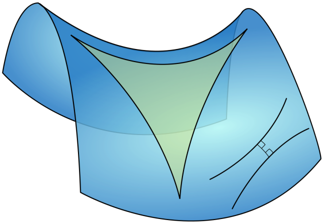


Sum of Interior Angles $= 180^\circ$ $< 180^\circ$

Models



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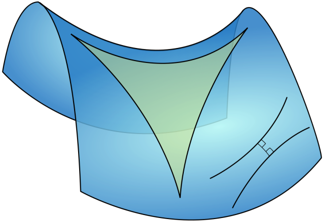

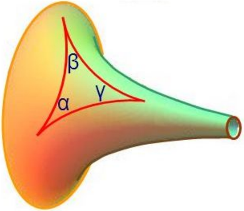

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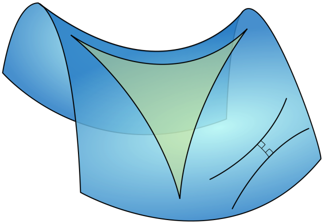

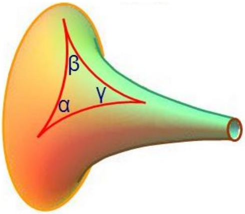

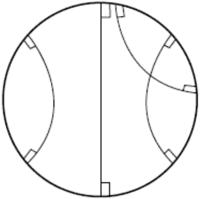

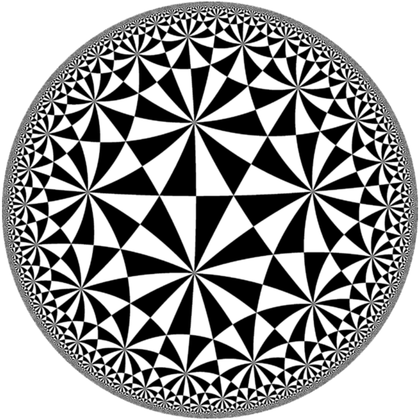

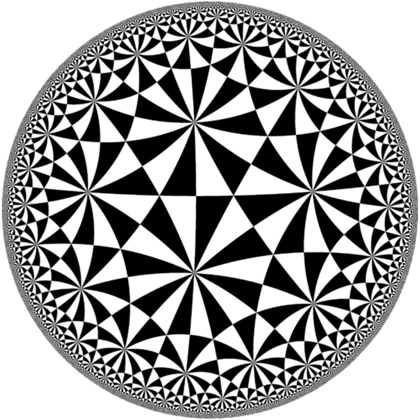

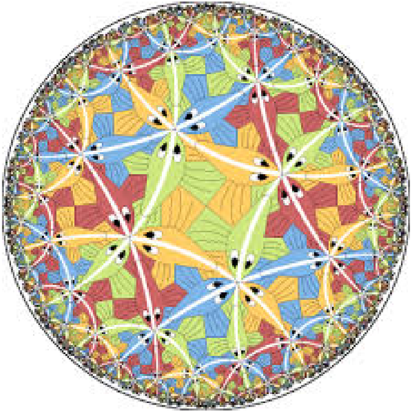

Another Non-Euclidean Geometry: Riemannian (or Elliptic) Geometry.

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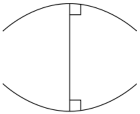
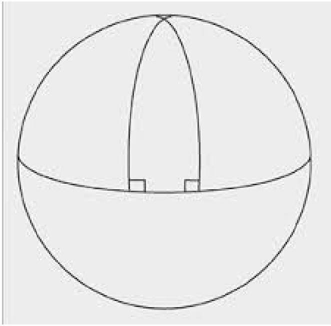
4. All right angles are equal to one another.

New Axiom: Given a line and a point not on it, there is *no* line parallel to the given line through the point.



Elliptic

Models



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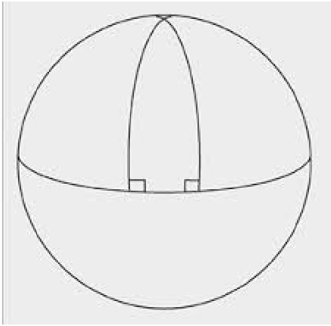

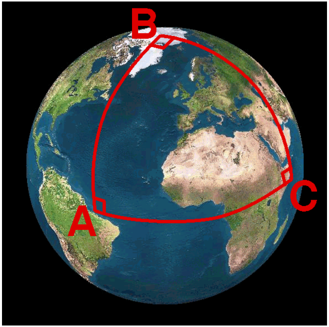

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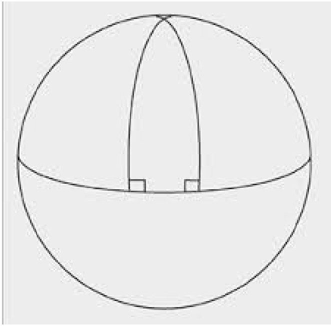
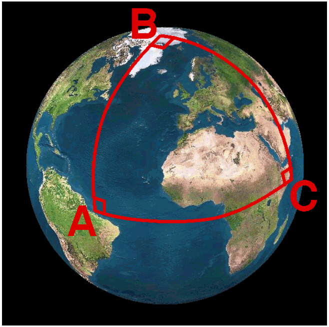
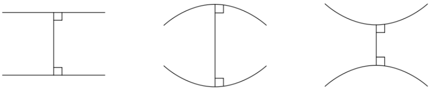
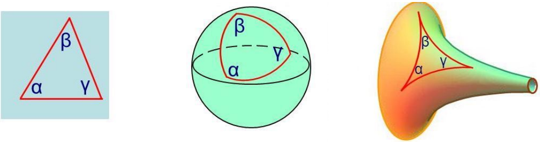


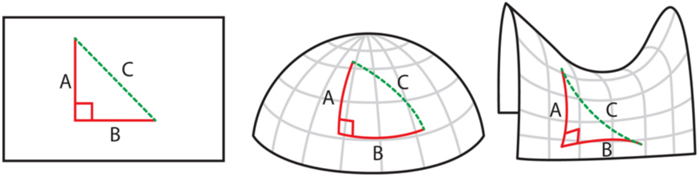




Euclidean Elliptic Hyperbolic



$\alpha + \beta + \gamma = 180^\circ$ $\alpha + \beta + \gamma > 180^\circ$ $\alpha + \beta + \gamma < 180^\circ$



Euclidean	**Spherical**	**Hyperbolic**
$a^2 + b^2 = c^2$	$\cos a \times \cos b = \cos c$	$\cosh a \times \cosh b = \cosh c$

The development of non-Euclidean geometry caused a profound revolution, not just in mathematics, but in science and philosophy as well. It changed the way mathematicians, scientists, and philosophers viewed their subjects.

Some geometers called Lobachevsky the "Copernicus of Geometry" due to the revolutionary character of his work

Non-Euclidean Geometries had several applications and paved the way for Einstein's General Theory of Relativity.

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It also led to efforts to axiomatize all of mathematics . . .

Zermelo-Fraenkel Axioms for Sets

Axiom 1: (The axiom of extension) Two sets are equal if and only if the have the same elements.

Axiom 2: (The axiom of the null set) There exists a set with no elements and we denote it by $\emptyset$.

Axiom 3: (The axiom of pairing) Given any sets A and B there exists a set whose elements are A and B.

Axiom 4: (The axiom of union) Given a set A, the union of all elements of A is a set

Axiom 5: (The axiom of the power set) Given any set A there exists a set containing all the subsets of A.

Axiom 6: (The axiom of separation) Given any set A and any sentence $p(x)$ that is a statement for all $x \in A$, then there is a set $\{x : p(x) \text{ is true}\}$.

Axiom 7: (The axiom of replacement) Given any set A and any function f defined on A, the image $f(A)$ is a set.

Axiom 8: (The axiom of infinity) There exists a set A such that $\emptyset \in A$ and whenever $B \in A$ it follows that $B \cup \{B\} \in A$.

Axiom 9: (The axiom of regularity) Given any nonempty set A, there exists $B \in A$ such that $B \cap A = \emptyset$.

The Axiom of Choice: Given any nonempty set A whose elements are pairwise disjoint nonempty sets, there exists a set B consisting of exactly one element taken from each set belonging to A.

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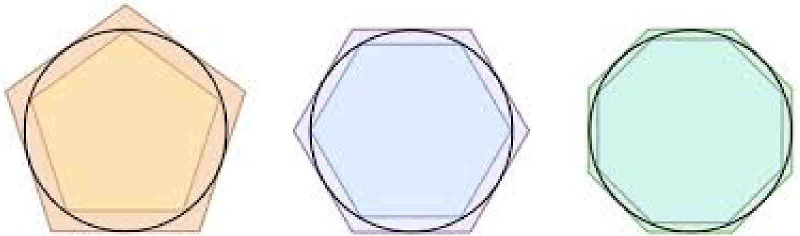
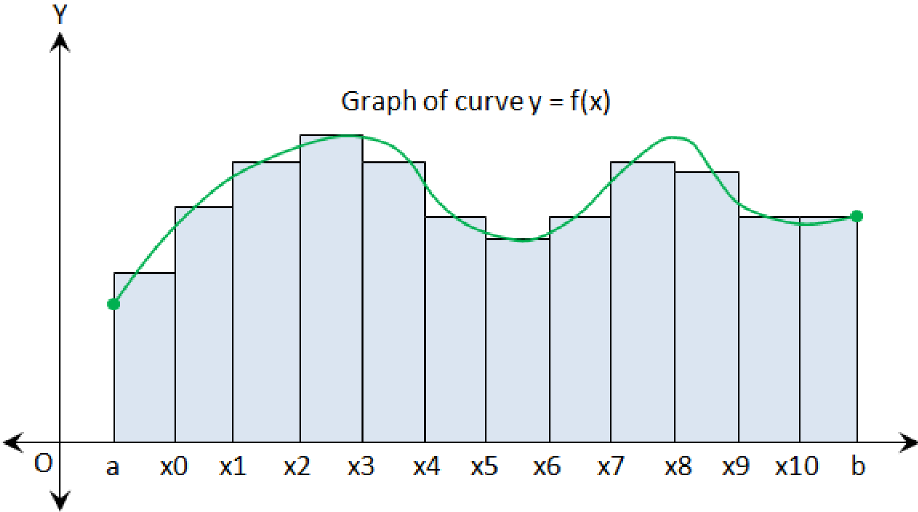
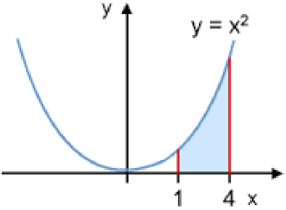
The Banach-Tarski Paradox

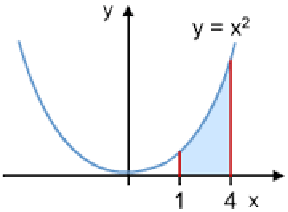
Archimedes' Method of Exhaustion



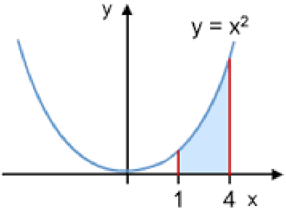
Riemann Integration



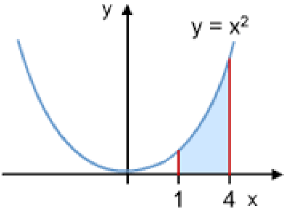

y
y = x²
1
4
x



$$\int_1^4 x^2\,dx$$



$$\int_1^4 x^2\,dx = \frac{1}{3}x^3 \,\Big|_1^4$$

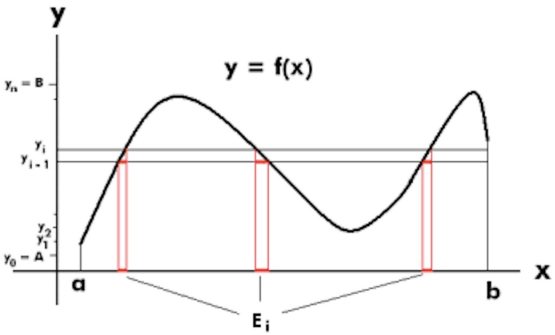


$$\int_1^4 x^2\,dx = \frac{1}{3}x^3 \,|_1^4 = \frac{1}{3}(4)^3 - \frac{1}{3}(1)^3 = 21$$

Works well for continuous functions, but is problematic for more complicated functions.

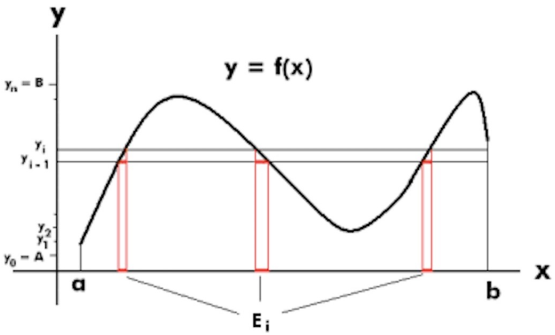
Henri Lebesgue, 1875–1941

French mathematician came up with a way to integrate more functions.



Area of "rectangle" = "length of base" × "height".

Need to find "length" for subsets of line. Mathematicians use the term "measure" instead of "length".



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Lebesgue's work showed that there is a large collection of subsets of the real line that you can assign a measure. (Results such as the Banach-Tarski paradox show it cannot be done for all.)

Some Russian History

1721–1917, Russian Empire
1917–1922, Russian Soviet Federative Socialist Republic
1922–1991, Soviet Union
1991–now, Russian Federation

1917, February Revolution, workers strike protesting lack of food, soldiers side with strikers, lasted less than a week, end of czardom, Provisional Government established.

1917, October Revolution (also called Red October, the October Uprising or the Bolshevik Revolution), the Bolsheviks, led by Vladimir Lenin, and the workers' Soviets overthrew the Provisional Government, change in Russia's social structure, paved way for Soviet Union.

November 1917 – October 1922, The Russian Civil War, multi-party war, many factions vied for power, including the Red Army, fighting for the Bolshevik form of socialism. Red Army ultimately victorious and Soviet Union established.

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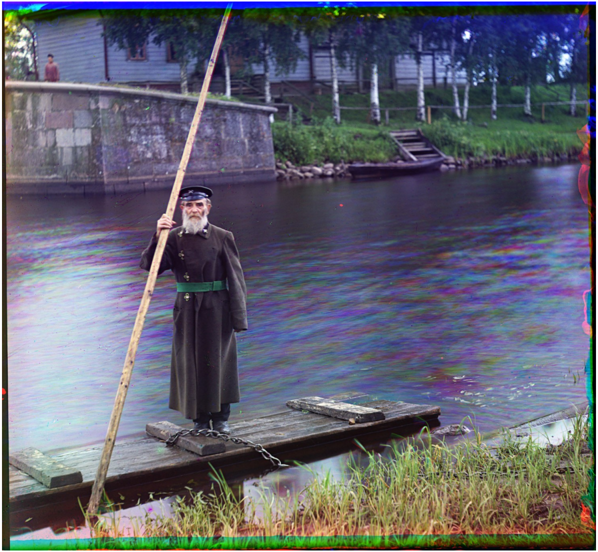
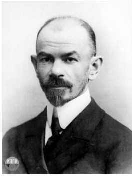
Vladimir Lenin, leader from 1917–1924
Served as head of government of the Russian Republic from 1917 to 1918, of the Russian Soviet Federative Socialist Republic from 1918 to 1922, and of the Soviet Union from 1922 to 1924. Lenin's government was led by the Bolsheviks, later renamed the Communist Party, with some powers initially also held by elected soviets.

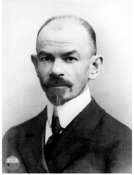
Joseph Stalin, leader from 1924–1953
Successor of Lenin, appointed General Secretary of the Communist Party of the Soviet Union in 1922. Leader of the Soviet Union from 1924 until his death in 1953. Effectively the dictator of the state. He managed to consolidate power after Lenin's death by suppressing Lenin's criticisms and expanding the functions of his role, all the while eliminating opposition.

The Sergei Mikhailovich Prokudin-Gorskii Collection features color photographic surveys of the vast Russian Empire made 1905–1915.





Dmitri Egorov, 1869–1931   Nikolai Luzin, 1883–1950

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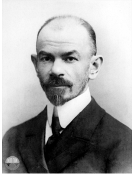
Egorov and his student Luzin are considered the founders of the Moscow School of Mathematics.



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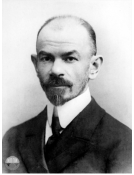

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Egorov held spiritual beliefs to be of great importance, and openly defended the Russian Orthodox Church against Marxist supporters after the Russian Revolution. In 1930 he was arrested and imprisoned as a "religious sectarian", and soon after was expelled from the Moscow Mathematical Society. Upon imprisonment, Egorov began a hunger strike that led to his death.

"It was Luzin whom the vision of the Moscow school of today had started with. Luzin was interested in foundations, and description for him was the method of understanding the whole of mathematics."

Started the subject of "Descriptive Set Theory". This subject seeks to "describe" those sets that can be assigned a measure.

Doctoral students included some of the most famous of Soviet mathematicians: Aleksandr Khinchin, Andrey Kolmogorov, Alexey Lyapunov, Pyotr Novikov, Mikhail Suslin, and Pavel Urysohn among others.

Luzin taught and nurtured a remarkable group of mathematicians endowed with overwhelming talent.

Luzitania — a group of students who followed Luzin's mathematical ideas.

Luzin (like Egorov) was also practicing member of Russian Orthodox Church but his behavior was more careful and his views were not demonstrated openly.

The Luzin Affair

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The very next day Pravda, on p. 2, has an unsigned article entitled "'On enemies in Soviet masks." It includes the following: "... the deliberate overpraising of these students is far from accidental. It is but one link in a long chain, and is very instructive in showing the methods by which enemies mask themselves. We well know that N. Luzin is an anti-Soviet person .... Academician Luzin could have become an honest Soviet scholar, as did many from the older generation. But he didn't want this; he, Luzin, remained an enemy, counting on ... the impenetrability of his mask.... It won't work, Gospodin Luzin!"

The anonymous article on July 3, accused Luzin of

1. praising weak work,
2. publishing his best papers in the West, and only second-rate ones in the USSR,
3. claiming his pupils' results as his own (in particular, those of Suslin and Novikov),
4. keeping good young candidates out of the Academy, and
5. continuing to hold reactionary ideas from Tsarist times. It also described Luzin as "an enemy in a Soviet mask".

Mekhlis, the editor of Pravda, immediately wrote to the Party's Central Committee, which authorized further inquiries, especially on item (ii).

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According to some researchers, Alexandrov and Kolmogorov may have been involved in a homosexual relationship, and there is a theory that both were blackmailed to accuse Luzin under threats to reveal this relationship.

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To prove the scientific misconduct of Luzin it was alleged that Luzin ingratiated and flattered H. Lebesgue by ascribing Luzin's sieve method to H. Lebesgue.

On the other hand, H. Lebesgue wrote in his preface to the Luzin book on analytic sets:
"Anyone will be astonished to find out from Luzin's book that I had incidentally invented the sieve method and was the first to construct an analytic set. But nobody could be more amazed than me. Mr. Luzin feels himself happy only when he has managed to ascribe his own discoveries to someone else."

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Although the Commission convicted Luzin, he was neither expelled from the Academy nor arrested. (Members of the academy receive a salary from the academy approximately equal to the salary of a professor, in addition to their salary from the university or institute where they work.) Subsequently Luzin worked in various institutes of the academy.

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Historian of mathematics A.P. Yushkevich speculated that at the time, Stalin was more concerned with forthcoming Moscow Trials of Lev Kamenev, Grigory Zinoviev, and others, and that the eventual fate of Luzin was of little interest to him.

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There was never any evidence (and there is still none today) that Luzin was guilty of any of the academic misdeeds of which he was accused. By all accounts he was an incredibly moral and honorable scholar.

The decision of the USSR Academy of Sciences against Luzin, adopted in 1936, has recently been reversed in 2012 by the same Academy, more than 60 years after his death.


Thank You!