Please do the following problems and hand them in with the other problems due on March 9.

Problem 1: (4 points) Define $T : \mathbb{R}^2 \to \mathbb{R}^4$ by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-3y \\ 0 \\ 4y \\ x+y \end{pmatrix}.$$

Prove that T is linear, and find the standard matrix for T.

Problem 2: (2 points) Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ yz \end{pmatrix}.$$

Prove that T is not linear.

Problem 3: (2 points) Prove that the set $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$ is a linearly independent subset of $\mathbb{R}^3$.

Problem 4: (2 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Prove that $\operatorname{Ran} T$ is a subspace of $\mathbb{R}^m$.