Zermelo-Fraenkel Axioms for Sets

Undefined terms: sets, membership (We shall think of the elements of sets as being sets themselves.)

Axiom 1: (The axiom of extension) Two sets are equal if and only if the have the same elements.

Axiom 2: (The axiom of the null set) There exists a set with no elements and we denote it by $\emptyset$.

Axiom 3: (The axiom of pairing) Given any sets A and B there exists a set whose elements are A and B.

Axiom 4: (The axiom of union) Given a set A, the union of all elements of A is a set

Axiom 5: (The axiom of the power set) Given any set A there exists a set containing all the subsets of A.

Axiom 6: (The axiom of separation) Given any set A and any sentence $p(x)$ that is a statement for all $x \in A$, then there is a set $\{x : p(x) \text{ is true}\}$.

Axiom 7: (The axiom of replacement) Given any set A and any function f defined on A, the image $f(A)$ is a set.

Axiom 8: (The axiom of infinity) There exists a set A such that $\emptyset \in A$ and whenever $B \in A$ it follows that $B \cup \{B\} \in A$.

Axiom 9: (The axiom of regularity) Given any nonempty set A, there exists $B \in A$ such that $B \cap A = \emptyset$.

In addition to the Zermelo-Fraenkel Axioms, there is one other axiom used in standard set theory:

The Axiom of Choice: Given any nonempty set A whose elements are pairwise disjoint nonempty sets, there exists a set B consisting of exactly one element taken from each set belonging to A.